

# Trade with R&D Costs to Entering Foreign Markets

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## Abstract

In this paper, I present a quality ladders endogenous growth model where firms differ in their productivities. I study the effect openness to trade has on firm productivity and firm turnover. Most theoretical papers in this literature assume an exogenous firm turnover rate. In this paper, the firm turnover rate is endogenously determined and in line with the empirical evidence, it depends on variable costs to trade. The paper is inspired by the theoretical work of Melitz (2003) and obtains Melitz-type results but with a different set of assumptions. In particular, I assume that firms invest in learning how to become exporters. I show that exporters are on average more productive than non-exporters and sell their products at higher prices. I also find that trade liberalization increases firm productivity and leads to a higher steady-state firm turnover rate, consistent with the empirical evidence.

Keywords: Trade liberalization, heterogeneous firms, endogenous turnover.  
JEL: F12, F13, F43, O31, O41.

## 1 Introduction

Up until several years ago, most of the endogenous growth literature that focused on trade-related issues modeled each firm as an exporter in addition to selling in its domestic market. But the evidence indicates that even in so-called export sectors, many firms do not export their products. The issue of which firms export is an important one and has been the topic of many recent papers in the trade literature. Research has concentrated on two factors

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by trade liberalization. This endogeneity comes naturally, since the model has a quality ladders structure. Firms do R&D to develop higher quality products, and when they succeed, they drive the previous quality leaders out of business. Innovation is associated with a process of creative destruction, as was originally emphasized by Schumpeter (1942). I show that trade liberalization (lowering the variable costs to trade) leads to an increase in the exit rate of firms. This result is consistent with the evidence in Pavcnik (2002), where it is reported that a period of trade liberalization in Chile (1979-1986) was accompanied by a "massive plant exit". Gibson and Harris (1996) have similar findings for New Zealand and Gu, Sawchuk and Rennison (2003) show a positive and increasing exit rate of firms as a result of tariff cuts in Canada c110(9)11(6810(9)11(6)7(1)11(9)11(6)10(9)10()-5075I)8(n)1469



This is a quality-augmented Dixit-Stiglitz consumption index, where  $d(j; !; t)$  denotes the quantity consumed of a product variety  $!$  of quality  $j$  at time  $t$ ,  $\lambda > 1$  is the size of each quality improvement and  $\sigma \in (0, 1)$  determines the elasticity of substitution between different products  $\frac{1}{1-\sigma} > 1$ .

Utility maximization follows three steps. The first step is to solve the within-variety static optimization problem. Let  $p(j; !; t)$  be the price of variety  $!$  with quality  $j$  at time  $t$ . Households allocate their budget within each variety by buying the product with the lowest quality-adjusted price  $p(j; !; t) = p^j$ . If two products have the same quality-adjusted price, I assume that consumers buy only the higher quality product. I will from now on write  $p(!; t)$ , to denote the price of the product within variety  $!$  with the lowest quality-adjusted price. Demand for all other qualities is zero.

The production of output is characterized by constant returns to scale. It takes one unit of labor to produce one unit of a good regardless of product quality. The wage rate is normalized to one and firms are price-setters. Each firm produces and sells a unique product  $i$ . Profits of a producer depend on what it sells domestically and abroad if it exports. An exporter needs to ship  $\tau > 1$  units of a good in order for one unit to arrive at the foreign destination. Let  $\pi_L(i; t)$  and  $\pi_E(i; t)$  denote profits from local sales and from exporting, respectively, of a company based at Home. Let  $D_L(i; t)$  denote demand for a product  $i$  in the Home country. Knowing that lower quality products can be produced by the competitive fringe, the profit-maximizing price that quality leaders can charge at home and abroad is the limit-price if  $\tau < 1$

### 2.3 R&D Races and the R&D Cost to Becoming an Exporter.

There are two R&D activities within this model described by two distinct R&D technologies: inventing higher quality levels of existing products and learning how to export. Labor is the only input used in both R&D activities. There are quality leaders, firms that hold the patent for the most advanced product within a certain product variety and follower firms, that try to improve the products that are sold by leaders. I solve for an equilibrium where Home firms do not improve on products originating from Foreign and Foreign firms do not improve on products originating from Home.

Leaders that produce for the local market do not try to improve on their own products. Given the same R&D technology as that of followers, they have a smaller incentive to innovate in comparison to followers. A non-exporting leader has strictly less to gain  $\pi_L(j+1) - \pi_L(j)$  from improving on its own product (omitting  $\beta$  and  $t$  for brevity) compared to a follower who would gain  $\pi_L(j+1)$ , hence leaders can not successfully compete for R&D financing with followers. If a leader is an exporter, the gain will be  $\pi_L(j+1) + \pi_E(j+1) - \pi_L(j) - \pi_E(j)$ . That gain is lower than that of a follower  $\pi_L(j+1)$  if  $\alpha < 2$  (as shown in the appendix). Given  $\beta < 1$ , for exporting leaders not to have an incentive to improve on their own products, I must have  $\alpha < 2^{1-\beta}$ . Limit pricing requires  $\alpha < 1$  and for firms to be able to export requires  $\alpha > \frac{1}{2}$ . Hence I can write my final assumption on  $\alpha$  as  $\frac{1}{2} < \alpha < \min\{1, 2^{1-\beta}\}$ . This guarantees that exporting leaders do not try to improve their own products.

Followers are the ones that invest in quality improving R&D and once they discover a state-of-the-art quality product, they take over the local market from the previous leader. Let  $I_i$  denote the Poisson arrival rate of improved products attributed to follower  $i$ 's investment in R&D. The innovative R&D technology for follower firm  $i$  is given by  $I_i = Q_t \frac{A_F I_i}{j^{(\beta/t)}}$ , where  $I_i$  is the labor input invested by the follower,  $\beta < 1$  is an R&D spillover parameter, and  $A_F > 0$  is an R&D productivity parameter. The R&D spillover parameter  $\beta$  is that can be positive or negative but the restriction  $\beta < 1$  is necessary to ensure that the model has a finite equilibrium rate of economic growth. The R&D technology available to followers takes into consideration the current development of the particular industry and requires more R&D effort in order to preserve the same Poisson arrival rate for higher quality products. The term  $j^{(\beta/t)}$  in the R&D technology captures the idea that it is more difficult to discover more sophisticated products and rules out any scale effects that would otherwise appear given the positive population growth rate. Followers targeting exporters have the same R&D technology as followers targeting non-exporters.

The returns to innovative R&D are independently distributed across firms, across product varieties and over time. Summing over all firms, I obtain that the Poisson arrival rate of improved products attributed to all followers' investment in R&D within a particular product variety  $j$  is given by

$$I_j = \sum_i I_i = Q_t \frac{A_F I_j}{j^{(\beta/t)}}.$$

The model will be solved for an equilibrium where the product innovation rate  $I_j$  does not vary between product varieties  $j$ .

The second R&D activity is that of leaders learning how to become exporters. This activity can be seen as learning to comply with foreign market regulations, establishing a distribution network, more generally, paying for the information needed to adapt to a less familiar environment. In essence, the investment each firm needs to make in R&D labor to learn to enter the foreign market is a type of fixed cost of market entry, a common feature in the heterogeneous firm literature. The fixed cost here is stochastic and firms with more sophisticated products need to invest more in order to achieve the same arrival rate of the knowledge on how to enter the foreign market. Leaders invest  $I_E$  units of labor in an R&D technology which makes them exporters with an instantaneous probability (or Poisson arrival rate)

$$I_E = Q_t \frac{A_E I_E}{j(t)} ; \quad (3)$$

where  $A_E$  is an R&D productivity parameter,  $\alpha < 1$  measures the degree of decreasing returns to R&D expenditure, and  $\beta$  is the same R&D spillover parameter. The term  $j(t)$  appears again in the learning-to-export technology and captures the idea that it is more difficult to learn how to export a more advanced product.

There are four types of firms that sell products within the Home country. First, there are Home leaders who export their products. The measure of product varieties produced by these firms is  $n_{LE}$ . Second, there are Home leaders who do not export their products. The measure of product varieties produced by these firms is  $n_{LN}$ . Third, there are competitive fringe firms. If a better version of a product is developed abroad and the new Foreign leader has not learned yet how to export this product, then the next lower quality version



on by a follower at home, which happens at the rate  $\lambda$ . The new leader takes over the home market and sells the better version there, whereas the older version is sold abroad at marginal cost. This channel is depicted by the upper middle arrow in Figure 1. The new incumbent at home needs to learn how to export in order to take over the foreign market.

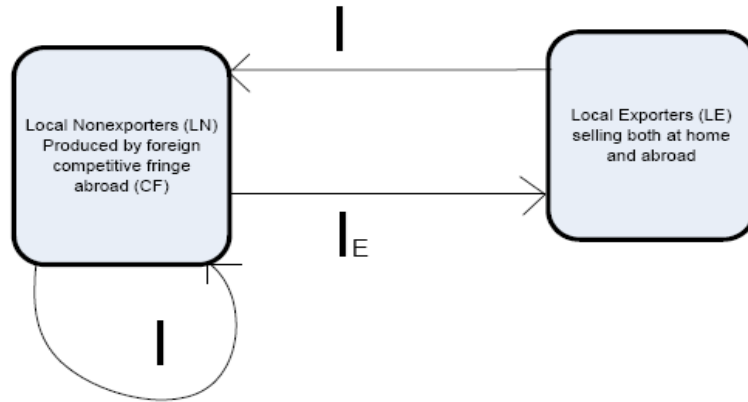


Figure 1. Product Dynamics

## 2.4 Bellman Equations and Value Functions

Firms maximize their expected discounted profits. Followers solve a stochastic optimal control problem with a state variable  $j(!; t)$ , which is a Poisson jump process of magnitude one. Non-exporting leaders maximize over the intensity of R&D dedicated to learning how to

The value of the firm increases in the quality of the product for which it holds a patent.

The Bellman equation for non-exporting leaders is given by:

$$rv_{LN}(j) = \max_{I_E} \left[ L(j) - I_E - I v_{LN}(j) + I_E (v_{LE}(j) - v_{LN}(j)) + \underline{v}_{LN}(j) \right] \quad (4)$$

This equation states that the maximized expected return on the non-exporting leader's stock must equal the return on an equal-sized investment in a riskless bond. The return is equal to a stream of profits minus investment in R&D for entering the foreign market, plus the arrival rates and respective changes in value attributed to being overtaken by a follower and becoming an exporter, plus the capital gain term  $\underline{v}_{LN}(j)$  because the value of the firm can change over time. Non-exporting leaders make a decision over  $I_E$ , how much to invest in R&D to learn how to export.

The Bellman equation for an exporting leader is simpler in the sense that exporting firms do not invest in R&D. They only exploit their quality advantage over other firms and the knowledge how to export. They face the risk of being removed by a firm that learns how to produce a higher quality version of the same product:  $rv_{LE}(j) = L(j) + I_E(j) - I v_{LE}(j) + \underline{v}_{LE}$ . The value of an exporting leader is derived from (4), after substituting for  $v_{LN}(j)$  and for  $I_E$  from (3). I obtain

$$v_{LE}(j) = \int_0^1 Q_t^{(j;t)} (I_E = (A_E) + 1 = A_F); \quad (5)$$

where <sup>1</sup>

2). The interpretation of the slope is that when R&D is relatively more difficult (higher  $x$ ), consumer demand must be higher to justify the higher R&D expenditures by firms.

Before moving on to find the labor equation, I need to first explore  $Q_t = \int_0^1 q(i;t) di$ , which is the average quality of all products sold in Home. This can be written as

$$Q_t = Q_{CF} + Q_{LE} + Q_{FE} + Q_{LN}; \quad (7)$$

where  $Q_{CF} = \int_{m_{CF}}^R q(i;t) di$  is a quality index of the products produced by the Home competitive fringe,  $Q_{FE} = \int_{m_{FE}}^R q(i;t) di$  is a quality index of the products produced by Foreign exporters,  $Q_{LE} = \int_{m_{LE}}^R q(i;t) di$  is a quality index of products produced by Home leaders that export, and  $Q_{LN} = \int_{m_{LN}}^R q(i;t) di$  is a quality index of products produced by Home leaders that do not export. All of these quality indexes change over time and could be written as  $Q_{CF}(t)$ ,  $Q_{FE}(t)$ , etc., but the  $t$  is omitted for brevity.

All labor in the Home country is fully employed in equilibrium and is divided between employment in the R&D sector  $L_R(t)$  and employment in the production sector  $L_P(t)$ . Thus  $L_t = L_P(t) + L_R(t)$  must hold for labor to be fully employed. I now solve for  $L_R(t)$  and  $L_P(t)$ .

Starting with  $L_P(t)$ , demand by Home consumers for a product sold by a Home leader is  $d(i;t)L_t = \frac{q(i;t)}{Q_t} y(t)L_t$ . Demand for an exported product sold abroad is  $d(i;t)L_t$ , but  $d(i;t)L_t$  needs to be shipped, hence  $\frac{q(i;t)}{Q(t)} y(t)L_t$  is produced. Demand for a product produced by the competitive fringe is  $\frac{q(i;t)}{Q_t} y(t)L(t)$ , where I multiply by  $\frac{1}{m_{CF}}$  to take into consideration that the competitive fringe prices at marginal cost, which is one. Thus, total production employment  $L_P(t)$  can be expressed as:

$$L_P(t) = \int_{m_{LE} + m_{LN}}^Z d(i;t)L_t di + \int_{m_{LE}}^Z d(i;t)L_t di + \int_{m_{CF}}^Z d(i;t)L_t di$$

At this stage, it is useful to define  $q_{LN} = \frac{Q_{LN}}{Q_t}$ ,  $q_{LE} = \frac{Q_{LE}}{Q_t}$  and  $q_{CF} = \frac{Q_{CF}}{Q_t}$ . Each  $q$  represents the quality share of a particular group of firms in the total quality index  $Q_t$ , where the share is determined not only by the average quality within the group but also by the measure of firms constituting the group. Substituting and simplifying gives

$$L_P(t) = (q_{LN} + q_{LE} + q_{LE} + q_{CF}) y(t)L_t$$

Next, I solve for R&D employment  $L_R(t)$ , using the R&D technologies for quality innovation and learning how to export, and keeping in mind that the innovation rate  $I$

Full employment of labor implies that  $L_t = L_P(t) + L_R(t)$ . Dividing both sides by  $L_t$ , I obtain the labor equation:

$$1 = (q_{LN} + q_{LE} + q_{LE} + q_{CF})$$





Taking logs and differentiating the above expression with respect to time gives the utility growth rate  $g = \frac{1}{\alpha} \frac{\dot{u}_t}{u_t} = \frac{1}{\alpha} \frac{\dot{Q}_t}{Q_t}$ , which after substituting for  $\frac{\dot{Q}_t}{Q_t} = n$  yields  $g = \frac{n}{1-\alpha}$ . The utility growth rate is proportionate to the population growth rate  $n$ . Since static utility  $u_t$  is proportional to consumer expenditure  $c_t$  and static utility increases over time only because  $Q_t$  increases,  $Q_t$  is a measure of the real wage at time  $t$ . Thus the real wage growth rate is the same as the utility growth rate and therefore  $g$  also represents the rate of economic growth in this model.

## 2.8 Average Qualities and Prices of Exporters and Non-exporters.

In order for the measures  $m_{LN}$  and  $m_{LE}$  to remain constant in steady-state equilibrium, it must be the case that the outflow of firms from  $m_{LN}$  is equal to the inflow of firms into  $m_{LN}$ , in other words  $m_{LN} l_E = m_{LE} l$ . Substituting for  $m_{LN}$  from  $m_{LN} + m_{LE} = \frac{1}{2}$  yields  $\frac{1}{2} m_{LE} l_E = m_{LE} l$ , from which it follows that  $m_{LE} = \frac{l_E=2}{l+l_E}$  and  $m_{LN} = \frac{l=2}{l+l_E}$ . The last two equations show that an increase in  $l_E$  leads to a decrease in the measure of products purchased from non-exporters  $m_{LN}$  and an increase in the measure of products purchased from exporters  $m_{LE}$ .

The average quality of exporting firms is given by  $Q_E = \frac{Q_{LE} + Q_{FE}}{m_{LE} + m_{FE}}$ . This can be written alternatively as  $Q_E = \frac{Q_{LE}}{m_{LE}}$ .

distribution is substantially shifted to the right (higher productivity) compared to the non-exporter productivity distribution, but at the same time there is a significant overlap in these distributions, meaning that there does not exist a threshold productivity value separating exporters from non-exporters.

A number of recent papers point out the correlation of export status with prices charged by firms. Kugler and Verhoogen (2008) use data from Colombia to compare output prices (what firms charge on their home markets) and export status of manufacturers, and find a positive relationship. Exporters charge higher prices. Hallak and Sivadasan (2009) also find a positive relationship, using Indian and U.S. data. In my model, exporters charge , which is larger than the average price of non-exporters, which is a convex combination of the price charged by non-exporting leaders and the price one charged by competitive fringe firms.

In addition, Iacovone and Javorcik (2008) show that producers that will export a particular product in the future charge a higher price at home on average two years before exporting starts. In my model



In the measure of product varieties  $m_{LN} + m_{LE}$  where there are Home quality leaders, Home innovation occurs at the rate  $I$  and results in the death of these firms. In the measure of product varieties  $m_{CF}$  where there is a Foreign non-exporting leader and a Home competitive fringe of producers, both Foreign innovation (which occurs at rate  $I$ ) and Foreign learning how to export (which occurs at rate  $I_E$ ) result in the death of the current Home producers.

Using  $m_{LE} = \frac{I_E}{I + I_E}$  and  $m_{LN} = \frac{I}{I + I_E} = m_{CF}$ , I can calculate the steady-state firm turnover rate as

$$N_D = \frac{2I(I + I_E)}{2I + I_E}$$

From  $\frac{\partial N}{\partial I} = 2I^2$

$D+$

order to obtain values of  $l_E$  that would satisfy the condition in Proposition 2:  $l > \frac{l_E}{2}$ . The parameter  $\alpha = 0.5$  describes the degree of decreasing returns to R&D in learning how to export. Finally, in order to obtain a 2% annual economic growth rate  $g = \frac{n-1}{1-\alpha}$ , I set  $\alpha = 0.1$ .

To solve the model, first I solve (11) for the steady-state equilibrium value  $df_E$  and then I solve simultaneously the R&D equation (6) and the labor equation (8) for the steady-state equilibrium values of  $x$  and  $y$ . In the labor equation (8),  $q_{LN}$ ,  $q_{LE}$  and  $q_{CF}$  are determined by equations (18), (19) and (20) in the appendix. The results obtained from solving the model numerically are reported in Table 1. In this table, I study how the steady-state equilibrium changes when  $\alpha$  is decreased, that is, when trade liberalization occurs. To guarantee that the equilibrium condition  $\alpha < 1.25$  holds, I only allow for  $\alpha$  values that are less than 1.25 in Table 1. For  $\alpha > 1.25$ , firms have no incentive to become exporters since they are not able to compete with the competitive fringe abroad.

	$q_{LN}$	$q_{LE}$	$q_{CF}$	$l_E$	$x$	$y$	$u$
1:24	0.7347	0.0298	0.2056	0.0089	18133	0.7947	1.14962
1:20	0.5031	0.0945	0.3078	0.0412	18173	0.7847	1.15312
1:15	0.4170	0.1420	0.2989	0.0747	18873	0.7856	1.27201
1:10	0.3719	0.1728	0.2825	0.1019	19835	0.7875	1.44846
1:05	0.3430	0.1943	0.2685	0.1242	20949	0.7895	1.67074
1:00	0.3227	0.2100	0.2572	0.1427	22160	0.7914	1.93511

Table 1. The Effects of Trade Liberalization ( $\alpha$ )

From Table 1, it is clear that trade liberalization monotonically increases the steady-state level of relative R&D difficulty  $x$ . Since relative R&D difficulty  $x(t) = \frac{Q_t^1}{L_t}$  only gradually adjusts over time and a new higher steady-state value means that along the transition path  $Q_t^1$  grows at a higher rate than  $L_t$ , trade liberalization must lead to a temporary increase in the innovation rate  $l(t)$ . Trade liberalization has no effect on the steady-state innovation rate  $l = n - [(1-\alpha)(1-\alpha)]$  but it does lead to a temporary increase in innovation by firms.

From Table 1, trade liberalization also increases the rate at which firms learn how to become exporters  $l_E$  and the quality share of Home exporters in the total quality index  $q_{LE} = \frac{Q_{LE}}{Q_t}$ . The intuition behind these properties is quite straightforward: decreasing the costs to trade leads to higher profits from exporting and increases the incentives firms have to learn how to export. Firms respond by devoting more resources to learning how to export and the quality share of exporters increases as a result.

When solving the model numerically, I can also study the effects of trade liberalization on aggregate productivity. For this model, a natural measure of productivity at time  $t$  is real output  $q_t L_t = P_t$  divided by the number of workers  $L_t$ , or  $q_t = \frac{P_t}{L_t}$ . In steady-state equilibrium, consumer expenditure  $c$  does not change over time but the quality-adjusted price index  $P_t$  decreases over time due to increases in the quality of products. Thus this measure of productivity  $c = \frac{P_t}{L_t}$  increases over time in steady-state equilibrium. Furthermore, Dixit and Stiglitz (1977) have shown that each consumer's static utility level  $u_t$  coincides with their real

consumption expenditure, that is,  $u_t = c_t = P_t$ . Thus measuring productivity in this model is equivalent to measuring the static utility level of the representative consumer.

To explore how trade liberalization affects productivity, I study its effects on consumer utility  $u_t$ . Rewriting  $x(t) = Q_t^{-1} L_t$  as  $Q_t^{-1} = (x L_t)^{-\frac{1}{1-\alpha}}$  and substituting in (12) gives the following expression:

$$u_t = y x^{\frac{1-\alpha}{1-\alpha}} L_t^{\frac{1-\alpha}{1-\alpha}} (q_{LN} + 2q_{LE})^{-\frac{1}{1-\alpha}} + q_{CF}^{-\frac{1}{1-\alpha}}$$

Since decreasing  $\alpha$  has no effect on  $L_t^{\frac{1-\alpha}{1-\alpha}}$ , I would like to see how

$$u = y x^{\frac{1-\alpha}{1-\alpha}} (q_{LN} + 2q_{LE})^{-\frac{1}{1-\alpha}} + q_{CF}^{-\frac{1}{1-\alpha}}$$

changes as  $\alpha$  decreases.

The results are reported in the final column of Table 1. It is clear that trade liberalization monotonically increases  $u$  and consequently, trade liberalization increases productivity at each point in time ( $\alpha \downarrow \Rightarrow u_t \uparrow$ ). The steady state utility growth rate  $g = \frac{n}{1-\alpha}$  depends only on the rate of population growth  $n$  and parameters  $\alpha$  and  $\beta$ . The part  $u$  of consumer utility that is affected by  $\alpha$  is constant over time in steady-state equilibrium. Thus trade liberalization influences the level of productivity at each point in time but not the long-run (or steady-state) productivity growth rate.

### 3 Conclusion

Following Melitz (2003), several models have developed the idea of firms with heterogeneous productivities in an endogenous growth setting. Most of them use a product variety expansion approach in their description of economic growth, whereas the current model analyzes trade liberalization in a quality ladder growth context. The literature shows that, under certain R&D parameter conditions and in line with empirical evidence, trade liberalization promotes productivity growth, by having less productive firms be replaced by more productive ones. I reach the same result but with a different set of assumptions.

Describing the process of becoming an exporter as a learning experience is the novel feature of this model. The knowledge how to successfully export involves investing in R&D and comes after an uncertain period of time. This divides firms into exporters and non-exporters but allows for the presence of large and relatively more productive non-exporting firms. The model does not generate a productivity threshold that cuts off exporters from non-exporters, which is a feature present in the other papers in the literature. In line with empirical evidence, exported products in the current model are more expensive, products that are to be exported are more expensive than products that are never intended for a foreign market, exporters are on average more productive and firm turnover is endogenous and depends on variable costs to trade.

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## Appendix

### Consumption

The second step of the static optimization problem is to find demand for product  $j$ :

$$\max_{d_j} Z = \sum_{j=0}^J u_j(d_j; t)$$

subject to  $c_t = \sum_{j=0}^J p_j(d_j; t) d_j$ , where  $j = 0, 1, \dots, J$

## Intertemporal Consumer Optimization

$$\begin{aligned} \ln u(t) &= \frac{1}{\alpha} \ln \int_0^1 q(j;t) p(j;t) c_t^\alpha dj \\ &= \ln c_t + \frac{1}{\alpha} \ln \int_0^1 q(j;t) p(j;t) dj \end{aligned}$$

Consumers make decisions over  $c_t$  and take the time paths of  $q(j;t)$ ,  $p(j;t)$ , and  $\int_0^1 q(j;t) p(j;t) dj$ , as well as the quality-adjusted price index term  $\int_0^1 q(j;t) p(j;t) dj$  as given. What remains to be solved is:  $\max \int_0^1 e^{-(n)t} \ln c_t dt$  subject to  $\dot{a}_t = w + (r_t - n)a_t - c_t$ . The last equation describes the intertemporal budget constraint of an individual consumer, where  $a_t$  is an individual asset holding and  $w$  is the wage per capita. The Hamiltonian becomes  $H = e^{-(n)t} \ln c_t + \lambda (w + (r_t - n)a_t - c_t)$ . The derivative with respect to consumption is  $H_c = \frac{e^{-(n)t}}{c_t} - \lambda = 0$ , from where I obtain  $\lambda = e^{-(n)t} c_t^{-1}$ . Taking logs and differentiating yields  $\dot{\lambda} = -n - \frac{\dot{c}_t}{c_t}$ . From the costate equation  $H_\lambda = \dot{\lambda} = -(r_t - n)$ , I obtain  $-\frac{\dot{c}_t}{c_t} = n - r_t$ . Combining the last two results gives the standard Euler equation  $\frac{\dot{c}_t}{c_t} = r_t - n$ .

## Condition for Exporting Leaders to Not Improve on Their Own Products

$$\begin{aligned} L(j+1;t) &> L(j;t) + E(j;t) - L(j;t) - E(j;t) \\ 0 &> E(j;t) - L(j;t) - E(j;t) \\ 0 &> ( \\ &0 \end{aligned}$$

Substituting for  $I_E$  from  $I_E = Q_t \frac{A_E I_E}{j(t)}$  and using ...rm pro...ts yields:

$$r v_{LN}(j) = (1 - j(t)) \frac{y(t)}{Q_t} L_t I_E^{\frac{1}{A_E}} \frac{j(t)}{Q_t} \\ I v_{LN}(j) + I_E (v_{LE}(j) - v_{LN}(j)) + \underline{v}_{LN}(j):$$

Then dividing by  $v_{LN}(j)$  and rearranging terms yields

$$r + I + \frac{Q_t - Q_t}{Q_t} = (1 - j(t)) \frac{y(t)}{v_{LN}(j) Q_t} L_t I_E^{\frac{1}{A_E}} + \frac{I_E}{v_{LN}(j) Q_t} \frac{j(t) I_E}{A_E}:$$

Finally, substituting for  $v_{LN}(j)$  and simplifying yields the R&D equation (6):

$$r + I + \frac{Q_t - Q_t}{Q_t} = (1 - j(t)) \frac{y(t)}{j(t) = A_F x(t)} \frac{j(t)}{j(t) = A_F} I_E^{\frac{1}{A_E}} + \frac{I_E}{j(t) = A_F} \frac{j(t) I_E}{A_E} \\ = (1 - A_F) \frac{y(t)}{x(t)} \frac{A_F}{A_E} I_E^{\frac{1}{A_E}} + \frac{A_F}{A_E} I_E^{\frac{1}{A_E}} \\ = (1 - A_F) \frac{y(t)}{x(t)} \frac{A_F}{A_E} I_E^{\frac{1}{A_E}} + \frac{A_F}{A_E} I_E^{\frac{1}{A_E}}$$

## Labor Equation

...n employment  $L_P(t)$  can be expressed as:



From  $Q_{LE} = Q_{LE} = (q_{LN} = q_{LE}) I_E$  and (9), I obtain

$$I = \frac{q_{LN}}{q_{LE}} I_E \frac{n}{1}:$$

From  $Q_{CF} = Q_{CF} = (q_{LE} = q_{CF}) I_E$  and (9),

**E**

(14)

Solving these equations using the quadratic formula, I obtain two solutions:

$$z_{1,2} = \frac{l_E + \frac{n}{1} \pm \sqrt{l_E^2 + 4l_E \left( \frac{n}{1} \right)}}{2l_E \left( \frac{n}{1} \right)}:$$

Expanding the expression under the square root, I obtain

$$\begin{aligned} & l_E + \frac{n}{1} \pm \sqrt{l_E^2 + 4l_E \left( \frac{n}{1} \right)} \\ = & l_E^2 + 2l_E \frac{n}{1} \pm \sqrt{2l_E^2 + \frac{n^2}{1} \pm 2 \frac{n}{1} \sqrt{2l_E^2 + (l_E)^2 + 4l_E \frac{n}{1} \pm 4l_E \frac{n}{1}}} \\ = & l_E^2 \pm 2l_E \frac{n}{1} \pm \sqrt{2l_E^2 + \frac{n^2}{1} \pm 2 \frac{n}{1} \sqrt{2l_E^2 + (l_E)^2}} \\ = & l_E \pm \frac{n}{1} \pm \sqrt{l_E^2}: \end{aligned}$$

It follows that the two solutions to the quadratic equation are:

$$z_{1,2} = \frac{l_E + \frac{n}{1} \pm \sqrt{l_E^2 + 4l_E \left( \frac{n}{1} \right)}}{2l_E \left( \frac{n}{1} \right)}$$

and since  $z$  must be positive to be economically meaningful, I can focus on the positive solution:

$$z = \frac{l_E + \frac{n}{1} + \sqrt{l_E^2 + 4l_E \left( \frac{n}{1} \right)}}{2l_E \left( \frac{n}{1} \right)}:$$

Simplifying, I obtain:

$$z = \frac{q_{LN}}{q_{LE}} = \frac{\frac{n}{1}}{8(a)10(i)6(n)12(:)JTJ/F22 11.9552 Tf 176.892 -21.957 Tq}$$

Finding  $I_E$

The Bellman equation for an exporting leader is

$$rV_{LE}(j) = L(j) + E(j) - I_{V_{LE}}(j) + \underline{V}_{LE}(j):$$

Next, substituting (18) into (16) and using  $\frac{1}{1}^n = I (1)$  yields

$$\begin{aligned}
 q_{CF} &= \frac{q_{LN} I_E \frac{q_{LE} \frac{1}{1}^n}{1} + I_E}{\frac{1}{1}^n + I_E} \\
 &= \frac{q_{LE} I \frac{q_{LE} \frac{1}{1}^n}{1} + I_E}{\frac{1}{1}^n + I_E} \\
 &= q_{LE} \frac{n}{(1)} \frac{\frac{1}{1}}{(\frac{1}{1}) + I_E} \\
 &= q_{LE} \frac{n}{(1)} \frac{\frac{1}{1}}{(\frac{1}{1}) + I_E} \\
 &= q_{LE} \frac{\frac{1}{1}^n}{\frac{1}{1}^n + I_E (1)}
 \end{aligned}$$

$$q_{CF} = q_{LE} \frac{I}{I (1) + I_E} \quad (19)$$

Substituting the above-derived expressions into (10) yields:

$$\frac{I}{I_E} q_{LE} + 2q_{LE} + \frac{I}{I (1) + I_E} q_{LE} = 1;$$

and then solving for  $q_{LE}$ , I obtain

$$q_{LE} = \frac{1}{\frac{I}{I_E} + 2 + \frac{I}{I (1) + I_E}} \quad (20)$$

Given  $I_E$  and  $I$ , equation (20) determines  $q_{LE}$ , then equation (19) determines  $q_{CF}$  and equation (18) determines  $q_{LN}$ .

It is possible to check that (10) holds:

$$\begin{aligned}
 q_{LN} + 2q_{LE} + q_{CF} &= \frac{1}{\frac{I}{I_E} + 2 + \frac{I}{I (1) + I_E}} \frac{I}{I_E} + \frac{2}{\frac{I}{I_E} + 2 + \frac{I}{I (1) + I_E}} + \\
 &+ \frac{1}{\frac{I}{I_E} + 2 + \frac{I}{I (1) + I_E}}
 \end{aligned}$$

## Finding the Utility Growth Rate

First, I note that

$$j(l;t) = (j(l;t))^{-\frac{1}{\sigma}} = q(l;t)^{-\frac{1}{\sigma}} = q(l;t)^{\frac{1}{\sigma-1}}$$

Using this result, substituting (2) into (1) yields

$$\begin{aligned} u_t &= \sum_{j=0}^{\infty} \frac{j(l;t) q(l;t) p(l;t) c_t}{P_t^{\frac{1}{\sigma}}} d!^{\frac{1}{\sigma-1}} \\ &= \sum_{j=0}^{\infty} \frac{j(l;t) q(l;t) p(l;t) y}{Q_t} d!^{\frac{1}{\sigma-1}} \\ &= \frac{y}{Q_t} \sum_{j=0}^{\infty} q(l;t)^{\frac{1}{\sigma-1}} q(l;t) p(l;t) d!^{\frac{1}{\sigma-1}} \\ &= \frac{y}{Q_t} \sum_{j=0}^{\infty} q(l;t) p(l;t) d!^{\frac{1}{\sigma-1}} \\ &= \frac{y}{Q_t} (Q_{LN} + 2Q_{LE})^{\frac{1}{\sigma-1}} + Q_{CF}^{\frac{1}{\sigma-1}} \\ &= y Q_t^{\frac{1}{\sigma-1}} (q_{LN} + 2q_{LE})^{\frac{1}{\sigma-1}} + q_{CF}^{\frac{1}{\sigma-1}} \end{aligned}$$

Taking logs and differentiating  $u_t$  with respect to time gives

$$g \frac{u_t}{u_t} = \frac{1}{\sigma-1} \frac{Q_t}{Q_t} = \frac{1}{\sigma-1}$$

## Average Quality of Exporters and Non-exporters

Under what conditions is the average quality of exporters higher than the average quality of non-exporters  $Q_E > Q_N$ ? For this inequality to hold, it must be the case that  $\frac{Q_E}{m_{LE}} > \frac{Q_N + Q_{CF}}{2m_{LN}}$ , which can be rewritten as  $\frac{2m_{LN}}{m_{LE}} > \frac{Q_N + Q_{CF}}{Q_E}$ . Using  $m_{LE} = \frac{l_E - 2}{l + l_E}$  and  $m_{LN} = \frac{l - 2}{l + l_E}$ , the LHS of this last inequality condition can be written as  $2 \frac{l - 2}{l + l_E} \frac{l + l_E}{l + l_E} = \frac{2l}{l + l_E}$ . The RHS can be transformed using (18) and (19) into  $l + l_E + l = (l + 1) + l_E$ . Thus, the question becomes, when does  $\frac{2l}{l + l_E} > l + l_E + l = (l + 1) + l_E$  hold? Multiplying both sides by  $\frac{l + l_E}{l} > 0$ , I obtain  $2 > (l + l_E) + l = (l + 1) + l_E$ . This inequality is equivalent to  $(l + 1)(2 - l) + l_E(2 - l) - l_E > 0$ , which simplifies to  $(l + 1)(2 - l) > l_E(l - 1)$ . The  $l$  terms cancel and I conclude that  $Q_E > Q_N$  holds if  $l > \frac{l + l_E}{2}$  and  $2 > l + l_E$ .