Micro to Macro: Optimal Trade Policy with Firm Heterogeneity

Arnaud Costinot
MIT

Andrés Rodríguez-Clare
UC Berkeley

Iván Werning
MIT

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Abstract

The empirical observation that "large firms tend to export, whereas small firms do not" has transformed the way economists think about the determinants of international trade. Yet, it has had surprisingly little impact about how economists think about trade policy. In this paper, we characterize optimal trade policy in a generalized version of the trade model with monopolistic competition and firm-level heterogeneity developed by Melitz (2003). At the micro-level, we find that optimal import taxes discriminate against the most profitable foreign exporters, while optimal export taxes are uniform across domestic exporters. At the macro-level, we demonstrate that the selection of heterogeneous firms into exporting tends to create aggregate nonconvexities that dampen the incentives for terms-of-trade manipulation, and in turn, the overall level of trade protection.

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we relax all of these assumptions, we derive new results about optimal trade taxes at the micro-level, and we generalize prior results about optimal trade taxes at the macro-level. Beside greater generality, these results uncover a novel connection between rm heterogeneity, aggregate nonconvexities, and lower levels of trade protection.

In terms of methodology, our analysis builds on the work of Costinot, Lorenzoni and Werning (2014) and Costinot, Donaldson, Vogel and Werning (2015) who characterize the structure of optimal trade taxes in a dynamic endowment economy and a static Ricardian economy, respectively. Like in the two previous papers, we use a primal approach and general Lagrange multiplier methods to characterize optimal wedges rather than explicit policy instruments. The novel aspect of our analysis is to break down the problem of nding optimal wedges into a series of micro subproblems, where we study how to choose micro-level quantities to deliver aggregate quantities at the lowest possible costs, and a macro problem, where we solve for the optimal aggregate quantities. The solutions to the micro and macro problems then determine the structure of optimal micro and macro taxes described above. This decomposition helps to highlight the deep connection between standard terms-of-trade argument, as in Baldwin (1948) and Dixit (1985), and the design of optimal trade policy in models of monopolistic competition.

In spite of their common rationale, i.e., terms-of-trade manipulation, the speci c policy prescriptions derived under perfect and monopolistic competition differ sharply. In Costinot, Donaldson, Vogel and Werning (2015), optimal export taxes should be heterogeneous, whereas optimal import tariffs should be uniform. This is the exact opposite of what we nd under monopolistic competition. In a Ricardian economy, goods exported by domestic rms could also be produced by foreign rms. This threat of foreign entry limits the ability of the domestic government to manipulate world prices and leads to lower export taxes on goods for which its rms have a weaker comparative advantage. Since the previous threat is absent under monopolistic competition, optimal export taxes are uniform instead. Conversely, lower import tariffs on the least pro table foreign rms under monopolistic competition derive from the existence of xed exporting costs, which are necessarily absent under perfect competition.

The previous discussion is related to recent results by Ossa (2011) and Bagwell and Staiger (2012b,a, 2015) on whether imperfectly competitive markets create a new rationale for the design of trade agreements. We hope that our analysis can contribute to the application of models with rm heterogeneity to study this question as well as other re-

distortions even within the same industry and opens up the possibility of terms-of-trade manipulation even at the rm-level. Our baseline analysis abstracts from these issues and instead focuses on the implication of the self-selection of heterogeneous rms into export markets, as in Melitz (2003). We come back to this point in our concluding remarks.

lated trade policy issues. Bagwell and Lee (2015) offer an interesting first step in that direction. They study trade policy in a symmetric version of the Melitz and Ottaviano (2008) model that also features the selection of heterogeneous firms into exporting. They show that this model provides a rationale for the treatment of export subsidies within the World Trade Organization.

The rest of the paper is organized as follows. Section 2 describes our basic environment. Section 3 sets up and solves the micro and macro planning problems of a welfare-maximizing country manipulating its terms-of-trade. Section 4 shows how to decentralize the solution to the planning problems through micro and macro trade taxes when governments are free to discriminate across firms. Section 5 studies the polar case where governments can only impose uniform taxes. Section 6 explores the sensitivity of our results to the introduction of multiple industries. Section 7 offers some concluding remarks.

2 Basic Environment

2.1 Technology, Preferences, and Market Structure

Consider a world economy with two countries, indexed by $i = H, F$; one factor of production, labor; and a continuum of differentiated goods or varieties. Labor is immobile across countries. w_i and L_i denote the wage and the inelastic supply of labor in country i, respectively.

Technology. Producing any variety in country i requires an overhead fixed entry cost, $f_i^e > 0$, in terms of domestic labor. Once the overhead fixed cost has been paid, firms randomly draw a blueprint $\varphi \in \Phi$. N_i denotes the measures of entrants in country i and G_i denotes the multivariate distribution of blueprints φ across varieties in that country. Each blueprint describes how to produce and deliver a unique variety to any country. $l_{ij}(q, \varphi)$ denotes the total amount of labor needed by a firm from country i with blueprint φ in order to produce and deliver $q \geq 0$ units in country j. We assume that

$$
\begin{aligned}
l_{ij}(q, \varphi) &= a_{ij}(\varphi) q + f_{ij}(\varphi), \text{ if } q > 0, \\
l_{ij}(q, \varphi) &= 0, \text{ if } q = 0.
\end{aligned}
$$

Technology in Melitz (2003) corresponds to the special case in which firms are heterogeneous in terms of productivity, but face constant iceberg trade costs, $a_{ij}(\varphi) \equiv \tau_{ij}/\varphi$, and constant fixed costs of selling in the two markets, $f_{ij}(\varphi) \equiv f_{ij}$.

Preferences. In each country there is a representative agent with a two-level homothetic utility function,

$$U_i = U_i(Q_{Hi}, Q_{Fi}),$$
$$Q_{ji} = [\;_{F} N_j (q_{ji}(j))^{1/m_j} dG_j(j)]^{m_j}.$$

where Q_{ji} denotes the subutility from consuming varieties from country j in country i, $q_{ji}(j)$ denotes country i's consumption of a variety with blueprint j produced in country j, and $m_j \; s_j/(s_j \; 1)$, with $s_j > 1$ the elasticity of substitution between varieties from country j. We do not restrict the elasticity of substitution between domestic and foreign goods. Melitz (2003) corresponds to the special case in which $m_H = m_F \; m$ and $U_i(Q_{Hi}, Q_{Fi}) \; [Q_{Hi}^{1/m} + Q_{Fi}^{1/m}]^m$.

Market Structure. All goods markets are monopolistically competitive with free entry. All labor markets are perfectly competitive. Foreign labor is our numeraire, $w_F = 1$.

2.2 Decentralized Equilibrium with Taxes

We focus on an environment in which governments have access to a full set of ad-valorem consumption and production taxes. We let taxes vary across markets and across rms.

We view the previous assumption as a useful benchmark. In theory, there is a priori no reason within the model that we consider why different goods should face the same taxes. In an Arrow-Debreu economy, imposing the same taxes on arbitrary subsets of goods would be ad-hoc. Changing the market structure from perfect to monopolistic competition does not make it less so. In practice, perhaps more importantly, different rms do face different trade taxes, ev8.9c4'GlTd [market st7, di do consumers through a lump-sum transfer, T_i.[4]

In a decentralized equilibrium with taxes, consumers choose consumption in order to maximize their utility subject to their budget constraint; rms choose their output in order to maximize their pro ts taking their residual demand curves as given; rms enter up to the point at which expected pro ts are zero; markets clear; and the government's budget is balanced in each country. Let $\overline{p}_{ij}(j)$ $m_i w_i a_{ij}(j)/(1 + s_{ij}(j))$ and $\overline{q}_{ij}(j)$ $[(1 + t_{ij}(j))\overline{p}_{ij}(j)/P_{ij}]^{s_i} Q_{ij}$. Using the previous notation, we can characterize a decentralized equilibrium with taxes as schedules of output, $\mathbf{q}_{\mathbf{j}}$ $f q_{ij}(j) g$, schedules of prices, $\mathbf{p}_{\mathbf{j}}$ $f p_{ij}(j) g$

2.3 Unilaterally Optimal Taxation

We assume that the government of country H, which we refer to as the home government, is strategic, whereas the government of country F, which we refer to as the foreign government, is passive. Namely, the home government sets ad-valorem taxes, $t_{HH} \quad \{ t_{HH}(j) \}$, $t_{FH} \quad \{ t_{FH}(j) \}$, $s_{HH} \quad \{ s_{HH}(j) \}$, and $s_{HF} \quad \{ s_{HF}(j) \}$, and a lump-sum transfer T_H in order to maximize home welfare, whereas foreign taxes are all equal to zero. This leads to the following de nition of the home government's problem.

De nition 1. The home government's problem is

$$\max_{T_H, \{t_{jH}, s_{Hj}\}_{j=H,F}, \{q_{ij}, Q_{ij}, p_{ij}, P_{ij}, w_i, N_i\}_{i,j=H,F}} U_H(Q_{HH}, Q_{FH}) \text{ subject to conditions (1)-(7).}$$

The goal of the next two sections is to characterize unilaterally optimal taxes, i.e., taxes that prevail at a solution to the domestic government's problem. To do so we follow the public nance literature and use the primal approach. Namely, we 9e7i narst239(appr)18(oach.)238(the

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the optimal micro and macro taxes, respectively, in the next section.

3.1 First Micro Problem: Producing Domestic Varieties

Consider the problem of minimizing the labor cost of producing Q_{HH} units of aggregate consumption for Home and Q_{HF} units of aggregate consumption for Foreign. This can be expressed as

$$L_H(Q_{HH},Q_{HF}) \equiv \min_{\tilde{q}_{HH},\tilde{q}_{HF},N} N\Big[\sum_{j=H,F}\int_{\mathcal{F}} l_{Hj}(\tilde{q}_{Hj}(\iota),\iota)\,dG_H(\iota) + f^e_H\Big] \quad (9a)$$

$$\int_{\mathcal{F}} N(\tilde{q}_{Hj}(\iota))^{1/\mu_H}\,dG_H(\iota) = Q_{Hj}^{1/\mu_H}, \text{ for } j = H,F. \quad (9b)$$

This minimization problem is infinite dimensional and non-smooth. Since there are fixed costs, the objective function is neither continuous nor differentiable around $q_{Hj}(\iota) = 0$ for any ι such that $f_{Hj}(\iota) > 0$. Given the additive separability of the objective and the constraint, however, it is easy to solve using a Lagrangian approach, as in Everett (1963).

The general idea is to proceed in two steps. First, we construct (q_{HH}, q_{HF}, N_H) that minimizes the Lagrangian associated with (9), given by $\mathcal{L}_H = N\ell_H$ where

$$\ell_H \equiv \sum_{j=H,F}\int_{\mathcal{F}} \Big[l_{Hj}(\tilde{q}_{Hj}(\iota),\iota) - \lambda_{Hj}(\tilde{q}_{Hj}(\iota))^{1/\mu_H}\Big]\,dG_H(\iota) + f^e_H.$$

Since, for given N, the Lagrangian is additively separable in $\{\tilde{q}_{Hj}(\iota)\}$, the optimization over these variables can be performed variety-by-variety and market-by-market. Although the discontinuity at zero remains, it is just a series of one-dimensional minimization problems that can be solved by hand. Second, we construct Lagrange multipliers, λ_{HH} and λ_{HF}, so that this solution satisfies constraint (9b) for $j = H,F$. By the Lagrangian Sufficiency Theorem, e.g. Theorem 1, p. 220 in Luenberger (1969), we can then conclude that the minimizer of $\mathcal{L}_H$ that we have constructed is also a solution to the original constrained minimization problem (9). [6]

For a given variety ι and a market j, consider the one-dimensional subproblem

$$\min_{\tilde{q}} l_{Hj}(\tilde{q},\iota) - \lambda_{Hj}\tilde{q}^{1/\mu_H},$$

[6] In general, a solution to the constrained minimization problem may not minimize the Lagrangian; see Sydsaeter (1974). Establishing the existence of a solution to the Lagrangian problem that satisfies (9b) is therefore a crucial part of the argument.

for an arbitrary Lagrange multiplier $\lambda_{Hj} > 0$. This leads to a simple cut-off rule

$$q_{Hj}(\omega) = \begin{cases} (\mu_H a_{Hj}(\omega)/\lambda_{Hj})^{\sigma_H}, & \text{if } \omega \in \Omega_{Hj}, \\ 0, & \text{otherwise,} \end{cases} \tag{10}$$

with $\Omega_{Hj} \equiv \{\omega : (\mu_H - 1)a_{Hj}(\omega)(\mu_H a_{Hj}(\omega)/\lambda_{Hj})^{\sigma_H} \geq f_{Hj}(\omega)\}$. Since L_H is linear in N the condition $\ell_H = 0$ is necessary and sufficient for an interior solution for N that satisfies (9b). Thus, the existence of a solution to the Lagrangian problem that satisfies (9b) reduces to finding $(\lambda_{HH}, \lambda_{HF}, N_H)$ that solves

$$\lambda_{Hj} = N_H\Big[\int_{\Omega_{Hj}} (\mu_H a_{Hj}(\omega))^{1-\sigma_H} dG_H(\omega)\Big]^{1/(1-\sigma_H)} Q_{Hj}^{1/\sigma_H}, \tag{11}$$

$$f_H^e = \sum_{j=H,F} \int_{\Omega_{Hj}} [(\mu_H - 1)a_{Hj}(\omega)(\mu_H a_{Hj}(\omega)/\lambda_{Hj})^{\sigma_H} - f_{Hj}(\omega)]dG_H(\omega). \tag{12}$$

A proof of existence and uniqueness is provided in Appendix A.1. This construction delivers a solution to problem (9). As shown in Appendix A.2, this solution must also be the unique solution to (9). We use this observation in the next section to establish necessary properties of optimal taxes.

By comparing equations (1), (2), (4), and (5), on the one hand, and equations (10), (11), and (12), on the other hand, one can check that conditional on Q_{HH} and Q_{HF}, the output levels and number of entrants in the decentralized equilibrium with zero taxes and the solution to the planning problem coincide. This reflects the efficiency of firm's level decision under monopolistic competition with Constant Elasticity of Substitution (CES) utility conditional on industry size; see Dixit and Stiglitz (1977) and Dhingra and Morrow (2012) for closed economy versions of this result. As shown in Section 4, this feature implies that the home government may want to impose a uniform import tariff or an export tax—in order to manipulate the fraction of labor allocated to domestic production rather than export—but that it never wants to impose taxes that vary across domestic firms, regardless of whether they sell on the domestic or foreign market.

3.2 Second Micro Problem: Importing Foreign Varieties

Let $P_{FH}(Q_{FH}, N_F)$ denote the minimum cost of one unit of aggregate imports at home conditional on total imports, Q_{FH}, as well as the measure of foreign entrants, N_F. Since foreign firms charge a constant markup over marginal cost and only enter home market if they can earn non-negative profits—condition (2) for N_F) a5-6 29 Tf 5.68577y718 Td [(.)-468(Since)]TJ -431.67

can be expressed as

$$P_{FH}(Q_{FH}, N_F) \quad \min_{\tilde{q}_{FH}} \quad {}_{F} N_F m_F a_{FH}(j) \tilde{q}_{FH}(j) dG_F(j) \tag{13a}$$

$$ {}_{F} N_F \tilde{q}_{FH}^{1/m_F}(j) dG_F(j) = 1, \tag{13b}$$

$$(m_F \quad 1) a_{FH}(j) Q_{FH} \tilde{q}_{FH}(j) \quad f_{FH}(j). \tag{13c}$$

This minimization problem is al5960((13c I.449 0 Td [tditively 0 Td separaal560(()-200(ith560(objective

The set Φ^c_{FH} will play a key role in our subsequent analysis. For varieties $\varphi \in \Phi^c_{FH}$, Home finds it optimal to alter its importing decision to make sure that foreign firms are willing to produce and export strictly positive amounts. This feature, which is at the core of models of trade with endogenous selection of firms into exporting, will lead to import taxes that vary across firms in Section 4.2.

Like in the Section 3.1, the final step of our Lagrangian approach consists in finding λ_{FH} such that constraint (13b), evaluated at $\{q_{FH}(\varphi)\}$, holds, that is

$$\int_{\Phi^u_{FH}} N_F(\mu_F^{2} a_{FH}(\varphi)/\lambda_{FH})^{1-\sigma_F} dG_F(\varphi) + \int_{\Phi^c_{FH}} N_F(f_{FH}(\varphi)/((\mu_F - 1)a_{FH}(\varphi)Q_{FH}))^{1/\mu_F} dG_F(\varphi) = 1. \quad (16)$$

The left-hand side is continuous, strictly increasing in λ_{FH}, with limits equal to zero and infinity when λ_{FH} goes to zero and infinity, respectively. By the Intermediate Value Theorem, there must therefore exist a unique λ_{FH} that satisfies (16), and, by the same argument as in Section 3.1, equations (15) and (16) characterize the unique solution to (13).

3.3 Macro Problem: Manipulating Terms-of-Trade

The goal of Home's planner is to maximize $U_H(Q_{HH}, Q_{FH})$ subject to the resource constraint (8) and the foreign equilibrium conditions. First, note that given the analysis of Section 3.1, the resource constraint can be expressed as

$$L_H(Q_{HH}, Q_{HF}) \leq L_H,$$

with $L_H(Q_{HH}, Q_{HF})$ given by (9). Second, note that the foreign equilibrium conditions can be aggregated into a trade balance condition. Conditions (3), (6), and (7) for $i = F$ imply that the value of Foreign imports must be equal to the value of its exports,

$$P_{HF}Q_{HF} = P_{FH}Q_{FH}.$$

Given the analysis of Section 3.2, we also know that the value of Foreign's exports must be equal to $P_{FH}(Q_{FH}, N_F)Q_{FH}$, with $P_{FH}(Q_{FH}, N_F)$ given by (13). In Appendix A.3, we show that using condition (1) for $i = F$ and $j = F$ and conditions (2)-(6) for $i = F$, we can also solve for the measure of foreign entrants, $N_F(Q_{FH})$, and the price of Home's exports, $P_{HF}(Q_{FH}, Q_{HF})$, as a function of aggregate exports and imports.

Combining the previous observations, we conclude that optimal aggregate quantities must solve the following macro problem,

$$\max_{Q_{HH},Q_{FH},Q_{HF}} U_H(Q_{HH}, Q_{FH}) \tag{17a}$$

$$P(Q_{FH}, Q_{HF})Q_{HF} = Q_{FH}, \tag{17b}$$

$$L_H(Q_{HH}, Q_{HF}) \quad L_H, \tag{17c}$$

where $P(Q_{FH}, Q_{HF}) \quad P_{HF}(Q_{FH}, Q_{HF})/ P_{FH}(Q_{FH}, N_F(Q_{FH}))$ denotes the price of Home's exports relative to its imports as a function of aggregate imports and exports. At this point, it should be clear that we are back to a standard terms-of-trade manipulation problem, with Home's planner internalizing the impact of its aggregate imports and exports, Q_{FH} and Q_{HF}, on its terms-of-trade, is the wedge that captures the terms-of-trade motive. Absent this motive, the only difference between MRS_H and P would be coming from the cost of producing Home's aggregate good for the domestic market relative to the foreign market, that is MRT_H. If there are no trade frictions, including no xed xed exporting costs, then $MRT_H = 1$ and equation (

While we have focused on domestic taxes, there is nothing in the previous proposition that hinges on domestic varieties being sold in the domestic market rather than abroad. Thus, we can use the exact same argument to characterize the structure of export taxes,

$s_{HF} \quad f s_{HF}$

This merely reflects our choice of benchmark variety for imports. t_{FH} is the tax on the variety j_{FH} such that the non-negativity constraint for foreigners' export profits is exactly binding at the unconstrained optimum: $q^u_{FH}(j_{FH}) = q^c_{FH}(j_{FH})$. We know from Lemma 3 that import taxes should be lower on varieties $j \in F^c_{FH}$. So in order to implement the same wedge, the domestic government must now impose import taxes on varieties $j \in F^u_{FH}$ that, relative to other taxes, are strictly greater than $1 + t$.

4.4 Implementation

Lemmas 1-4 provide necessary conditions that linear taxes have to satisfy so that the decentralized equilibrium replicates the first-best allocation. In the next lemma, which is proven in Appendix B.2, we show that that if the previous taxes are augmented with prohibitive taxes on the goods that are not consumed, $j \notin F_{HH}$, $j \notin F_{HF}$, and $j \notin F_{FH}$, then they are also sufficient to implement the first-best allocation.

Lemma 5. *There exists a decentralized equilibrium with taxes that implements the first-best allocation.*

Since Home's planning problem is a relaxed version of Home's government problem introduced in Definition 1 while setting the overall level other taxes such that $t_{HH} = s_{HH} = t_{FH} = 0$. This is an expression of Lerner symmetry, which must still hold under monopolistic competition. In this case, all varieties $j \in F^{c}_{FH}$ would receive an import subsidy equal to $q_{FH}(j) - 1 < 0$. As alluded to in Section 3.1, the fact that domestic taxes can be dispensed with derives from the efficiency of the decentralized equilibrium with monopolistic competition and CES utility. Here, as in Bhagwati (1971), trade taxes are the first-best instruments to exploit monopoly and monopsony power in world markets. We come back to this issue in Section 6 when discussing how our results extend to environments subject to home-market effects where the decentralized equilibrium is no longer efficient.

4.5 How Does Firm Heterogeneity Affect Optimal Trade Policy?

Using Proposition 1, we can take a first stab at describing how firm heterogeneity affects optimal trade policy. There are two broad insights that emerge from our analysis.

The first one is that macro-elasticities, h_{HF} and h_{FH}, determine the wedge, t, between Home and Foreign's marginal rates of substitution at the first-best allocation and, in turn, the overall level of trade protection, as established by condition (26). In line with the equivalence result in Arkolakis, Costinot and Rodríguez-Clare (2012), this is true regardless of whether or not firms are heterogeneous and only the most profitable ones select gravity models, like Anderson and Van Wincoop (2003) and Eaton and Kortum (2002), are equivalent to endowment models in which countries directly exchange labor services. Hence, conditional on the elasticity of their labor demand curves, the aggregate implications of uniform changes in trade costs, i.e. exogenous labor demand shifters, must be the same in all gravity models. The previous observation, however, does not imply that optimal policy should be the same in all these models. To the extent that optimal trade taxes are heterogeneous across goods, they will not act as simple labor demand shifters, thereby breaking the equivalence in Arkolakis, Costinot and Rodríguez-Clare (2012). This is what Proposition 1 establishes in the context of a canonical model of trade with monopolistic competition and firm-level heterogeneity à la Melitz (2003).

This general conclusion notwithstanding—micro-structure matters for optimal policy, even conditioning on macro-elasticities—it is worth noting that the specific policy prescriptions derived under perfect and monopolistic competition differ sharply. In Costinot, Donaldson, Vogel and Werning (2015), optimal export taxes should be heterogeneous, whereas optimal import tariffs should be uniform. This is the exact opposite of what conditions (23) and (25) prescribe under monopolistic competition. In a Ricardian economy, goods exported by domestic firms could also be produced by foreign firms. This threat of entry limits the ability of the home government to manipulate prices and leads to lower export taxes on "marginal" goods. Since this threat is absent under monopolistic competition, optimal export taxes are uniform instead. On the import side, lower tariffs on "marginal" goods under monopolistic competition derive from the existence of fixed exporting costs, which are necessarily absent under perfect competition.

5 Optimal Uniform Taxes

In previous sections, we have characterized optimal trade policy under the assumption that the home government is not only free to discriminate between firms from different countries by using trade taxes, but also unlimited in its ability to discriminate between firms from the same country. While this provides a useful benchmark to study the normative implications of firm heterogeneity for trade policy, informational or legal constraints may make this type of taxation infeasible in practice. Here, we turn to the other polar case in which the home government is constrained to set uniform taxes: $t_{HF}(j\ ) = \bar{t}_{HF}$, $t_{HH}(j\ ) = \bar{t}_{HH}$, $s_{HF}(j\ ) = \bar{s}_{HF}$, and $s_{HH}(j\ ) = \bar{s}_{HH}$ for all j .

5.1 Micro to Macro Once Again

To solve for optimal uniform taxes, we can follow the same approach as in Sections 3 and 4. The only difference is that the micro problems of Sections 3.1 and 3.2 should now include an additional constraint:

$$q_{ij}(j')/q_{ij}(j) = \left(a_{ij}(j')/a_{ij}(j)\right)^{s_F} \text{ for any } j, j' \text{ such that } q_{ij}(j'), q_{ij}(j) > 0. \quad (27)$$

can therefore show that optimal uniform taxes must satisfy

$$\frac{(1+\bar{t}_{FH})/\ (1+\bar{t}_{HH})}{(1+\bar{s}_{HF})/\ (1+\bar{s}_{HH})} = 1 + t \ . \tag{29}$$

Compared to the analysis of Section 4, the optimal wedge, $t = (h_{HF} + h_{FH})/\ (1 + h_{HF})$, stills depends exclusively on the terms-of-trade elasticities. The only difference is that the import price index that determines these elasticities is now given by equation (28).

In order to help our results to those in the existing literature, we set domestic and export taxes to zero in the rest of this section: $\bar{t}_{HH} = \bar{s}_{HH} = \bar{s}$ 28).

both homogeneous of degree zero,[7] equations (31) and (32) imply

$$e = 1/(d \ln MRS_F(Q_{HF}/Q_{FF}, 1)/d \ln(Q_{HF}/Q_{FF})), \quad (34)$$

$$k = 1/(d \ln MRT_F(Q_{FH}/Q_{FF}, 1)/d \ln(Q_{FH}/Q_{FF})). \quad (35)$$

By equations (31) and (32

Proposition 2. *Optimal uniform tariffs are such that*

$$\bar{t}_{FH} = \frac{1 + (e / k\ )}{(e \quad 1)x_{FF}}, \tag{39}$$

where e , k , *and* x_{FF} *are the values of* e, k, *and* x_{FF} *evaluated at those taxes.*

Equation (39) is a strict generalization of the optimal tariff formula derived under monopolistic competition by Gros (1987), cost. Indeed, the difference between the foreign rms' prices and their marginal costs is equal to $m_F \quad 1 = 1/(s_F \quad 1)$, which is the optimal tariff that a small open economy would choose when $e = s_F$. By allowing the upper-level elasticity of substitution, e, to differ from the lower-level elasticities of substitution, s_H and s_F, our analysis suggests that the rst of these two interpretations is the most robust. When $e \neq s_F$, foreign rms still charge a markup $m_F = s_F/(s_F \quad 1)$ on the goods that they export. Yet, the only relevant elasticity in this case is e because it is the one that shapes Home's terms-of-trade elasticities, as shown in equations (37) and (38). We come back to this issue in Section 6.3.

As noted above, Proposition 2 also generalizes the results of Demidova and Rodríguez-Clare (2009) and Felbermayr, Jung and Larch (2013) who focus on an economy à la Melitz (2003). Compared to the present paper, they assume a constant elasticity of substitution between domestic and foreign goods, $e = s_H = s_F \quad s$. They also assume that taxes are uniform across rms, that rms only differ in terms of their productivity, and that the distribution of rm-level productivity is Pareto. Under these assumptions, the decentralized equilibrium with taxes can be solved in closed-form. As discussed in Feenstra (2010), models of monopolistic competition with Pareto distributions lead to an aggregate production possibility frontier with constant elasticity of transformation,

$$k = \frac{sn \quad (s \quad 1)}{n \quad (s \quad 1)} < 0, \tag{40}$$

where $n > s \quad 1$ is the shape parameter of the Pareto distribution; see Appendix C.3.[9] Combining equations (39) and (40) and imposing $e = s$, we obtain

$$\bar{t}_{FH} = \frac{1}{(nm \quad 1)x_{FF}} > 0,$$

as in Felbermayr, Jung and Larch (2013). In the case of a small open economy, the previous expression simpli es further into $1/(nm \quad 1)$, as in Demidova and Rodríguez-Clare (2009).

[9]In his analysis of models of monopolistic competition with Pareto distributions, Feenstra (2010) concludes that rm heterogeneity leads to strictly convex production sets. In contrast, equation (40) implies that Foreign's production set is non-convex: $k < 0$. Both results are mathematically correct. The apparently opposite conclusions merely re ect the fact that we have de ned the aggregate production possibility frontier abroad as a function of the CES quantity aggregates, Q_{FH} and Q_{FF}, whereas Feenstra (2010) de nes them, using our notation, in terms of Q_{FH}^{1/m_F} and Q_{FF}^{1/m_F}.

5.4 Firm Heterogeneity, Aggregate Nonconvexities, and Trade Policy

Since $n > s \quad 1$, an intriguing implication of the results in Demidova and Rodríguez-Clare (2009) and Felbermayr, Jung and Larch (2013) is that conditional on $e \quad = s$ and x_{FF}, the optimal level of trade protection is lower when only a subset of rms select into exports than when they all do, $1/ \quad ((nm \quad 1)x_{FF}) < 1/ ((s \quad 1)x_{FF})$. This speci c parametric example, however, is silent about the nature and robustness of the economic forces leading up to this result.

Our general analysis isolates aggregate nonconvexities as the key economic channel through which rm heterogeneity tends to lower the overall level of trade protection. Mathematically, the previous observation is trivial. From equations (20) and (37), we know that $e \quad 1 > 0$. Since $k \quad ! \quad ¥$ when rms are homogeneous, we arrive at the following corollary of Proposition 2.

Corollary 1. *Conditional on* $(e \ , x_{FF})$, *optimal uniform tariffs are strictly lower with than without firm heterogeneity if and only if firm heterogeneity creates aggregate nonconvexities,* $k \quad < 0$.

Economically speaking, Home's trade restrictions derive from the negative effects of exports and imports on its terms of trade. By reducing the elasticity of Home's terms of trade with respect to its imports, in absolute value, aggregate nonconvexities dampen this effect, and in turn, reduce the optimal level of trade protection.

The nal question that remains to be addressed is how likely it is that the selection of heterogeneous rms into exporting will lead to aggregate nonconvexities. It is instructive to consider rst a hypothetical situation in which the measure of foreign rms, N_F, is exogenously given. In that situation, the selection of heterogeneous rms would necessarily lead to aggregate nonconvexities. To see this, note that equation (32) implies

$$MRT_F = \frac{(\quad _{F_{FH}} (a_{FH}(j \ ))^{1 \quad s_F} dG_F(j \ ))^{1/ (1 \quad s_F)}}{(\quad _{F_{FF}} (a_{FF}(j \ ))^{1 \quad s_F} dG_F(j \ ))^{1/ (1 \quad s_F)}},$$

with the set of foreign varieties sold in market $j = H, F$ such that

$$F_{Fj} = f j \ : (m_F$$

And since labor market clearing requires Q_{FF} to be decreasing in Q_{FH}, this implies that $MRT_F(Q_{FH}, Q_{FF}(Q_{FH}))$ is decreasing in Q_{FH}, i.e. that there are aggregate nonconvexities.

Intuitively, an increase in foreign exports, Q_{FH}, has two effects. First, it expands the set of foreign rms that export, which lowers the unit cost of Foreign's exports. Second, it lowers Q_{FF}, which reduces the set of foreign rms that sell domestically and raises the unit cost of Foreign's domestic consumption. Both effects tend to lower Foreign's opportunity cost of exports in terms of domestic consumption.

Our next result provides suf cient conditions such that the previous selection forces dominate any additional effect that changes in aggregate exports, Q_{FH}, may have on the number of foreign entrants, N_F, and in turn, the monotonicity of MRT_F. Let $N_F(Q_{FH}, Q_{FF})$

Proposition 3. *If the measure of foreign entrants increases with aggregate output to any market, then conditional on* (e , x_{FF})*, optimal uniform tariffs are lower with than without firm heterogeneity, with strict inequality whenever selection is active in at least one market.*

The active selection of heterogeneous rms may actually lower the overall level of trade protection so much that the optimal uniform tariff may become an *import subsidy*. To see this, note that as e goes to in nity, the optimal uniform tariff in equation (39) converges towards

$$\bar{t}_{FH} = 1/\,(\mathrm{k}\ x_{FF}),$$

which is strictly negative if there are aggregate nonconvexities abroad, k < 0.

imports, Q^k_{FH}, and the measure of foreign entrants, N^k_F,

$$P^k_{FH}(Q^k_{FH}, N^k_F) \quad \min_{q_{FH}} \quad {}_F \; N^k_F m^k_F a_{FH}(j\,)\, q_{FH}(j\,)\, dG^k_F(j\,)$$

$$_F \; N^k_F q_{FH}^{1/\, m^k_F}(j\,)\, dG^k_F(j\,) \quad 1,$$

$$m^k_F a_{FH}(j\,) Q^k_{FH} q_{FH}(j\,) \quad l_{FH}(Q^k_{FHq}$$

of its representative agent subject to a trade balance condition and a resource constraint,

$$\max_{Q_{HH},Q_{FH},Q_{HF}} U_H(U_H^1(Q_{HH}^1, Q_{FH}^1), ..., U_H^K(Q_{HH}^K, Q_{FH}^K)) \tag{41a}$$

$$\sum_k P_{FH}^k(Q_{FH}, Q_{HF})Q_{FH}^k \quad \sum_k P_{HF}^k(Q_{HF}, Q_{HF})Q_{HF}^k, \tag{41b}$$

$$\sum_k L_H^k(Q_{HH}^k, Q_{HF}^k) \quad L_H. \tag{41c}$$

Using the associated rst-order conditions, one can check that Home's planner would restrict all taxes to be uniform within the same sector, as in Section 5. This is equivalent to adding the sector-level counterpart of constraint (27) to the sector-level micro problems in Section 6.1. We let $\bar{t}^D_{HH}$, $\bar{t}^D_{FH}$, $\bar{s}^D_{HH}$, and $\bar{s}^D_{HF}$ denote the uniform ad-valorem taxes in the differentiated sector and $\bar{t}^O_H$ denote the ad-valorem trade tax-cum-subsidy in the homogeneous sector.[14]

In the outside sector, we assume that s^O_F ! ¥, that there are no xed costs of production and no trade costs, and that all rms at home and abroad have the same productivity, which we normalize to one. So, one can think of the homogeneous good as being produced by perfectly competitive rms in both countries. In the rest of this section, we use the outside good as our numeraire. As in the previous sections, we impose no restriction on the distributions of rm-level productivity and xed costs in the differentiated sector, G^D_H and G^D_F, nor on the sector-level aggregator, U^D_H and U^D_F, which determines the substitutability between domestic and foreign varieties in both countries. Finally, we let b_F denote the share of expenditure on differentiated goods in Foreign. Given our Cobb-Douglas assumption, this share is constant.

Let $X^O_H \quad Q^O_{FH} \quad Q^O_{HF}$ denote Home's exports of the outside good. Under the previous assumptions, Home's macro planning problem reduces to

$$\max_{Q^O_H, X^O_H Q^D_{HH}, Q^D_{FH}, Q^D_{HF}} U_H(Q^O_H \quad X^O_H, U^D_H(Q^D_{HH}, Q^D_{FH}))$$

$$P^D_{FH}(X^O_H, Q^D_{FH})Q^D_{FH} \quad P^D_{HF}(X^O_H, Q^D_{FH}, Q^D_{HF})Q^D_{HF} + X^O_H,$$

$$Q^O_H + L^D_H(Q^D_{HH}, Q^D_{HF}) \quad L_H,$$

with Home's import and export prices in the differentiated sector, $P^D_{FH}(X^O_H, Q^D_{FH})$ and $P^D_{HF}(X^O_H, Q^D_{HF}, Q^D_{FH})$, such that

$$P^D_{FH}(X^O_H, Q^D_{FH}) = m^D_F L^D_{FH}(Q^D_{FH}, Q^D_{FF}(Q^D_{FH}, L^D_F(X^O_H))),$$

$$P^D_{HF}(X^O_H, Q^D_{HF}, Q^D_{FH}) = m^D_F L^D_{FF}(Q^D_{FH}, Q^D_{FF}(Q^D_{FH}, L^D_F(X^O_H))) MRS^D_F(Q^D_{HF}, Q^D_{FF}(Q^D_{FH}, L^D_F(X^O_H))),$$

where L^D_{F} D fientiated sector and 0978 3.81 Tf

$Q^D_{FF}(Q^D_{FH}, L^D_F(X^O_H)))$, not only depends on foreign exports of the differentiated good, Q^D_{FH}, but also on the total amount of labor allocated to the differentiated sector, $L^D_F(X^O_H)$, which now appears as a second argument. Given Cobb-Douglas preferences, this only depends on Home's net imports of the outside good. Since Foreign always spends $(1 \quad b_F)L_F$ on the outside good, the amount of labor allocated to that sector must be equal to $(1 \quad b_F)L_F \quad X^O_H$ and the amount allocated to the differentiated sector must be equal to L_F minus this number, $L^D_F(M^O_H) = b_F L_F + X^O_H$.

In spite of the introduction of an outside sector, the relative price of Home's exports in the differentiated sector, $P^D \quad P^D_{HF}/ P^D_{FH}$, still satis es $P^D = MRS^D_F / MRT^D_F$. Since there are now three aggregate goods that are traded internationally—Home's and Foreign's

The previous wedges, in turn, pin down the relative level of optimal trade taxes. Using the same argument as in Section 4, one can show that

$$\frac{(1+\bar{t}^{D}_{FH})/(1+\bar{t}^{D}_{HH})}{(1+\bar{s}^{D}_{HF})/(1+\bar{s}^{D}_{HH})} = 1+t^{D}, \qquad (44)$$

$$(1+\bar{t}^{D}_{FH})/(1+\bar{t}^{O}_{H}) = 1+t^{O}. \qquad (45)$$

good, this creates a first improvement in its terms of trade. In addition, an increase in either imports of the differentiated good or exports of the homogeneous good raises foreign production of the differentiated good for its local market. Since P^D µ $P^D_{HF}/ P^D_{FF} = MRS^D_F$ in the absence of selection, this must be accompanied by a decrease in the relative price of Foreign's differentiated goods relative to Home's differentiated goods, a second improvement in Home's terms of trade. [16] When Home is a small open economy in the sense that $r^D_{FF} = 1$, it cannot manipulate entry or output abroad, which leads to zero subsidies: $(1 + \bar{t}^D_{FH})/ (1 + \bar{t}^O_H) = 1$. The same is true when s^D_F goes to infinity. In this case, the relative price of Foreign's differentiated goods relative to the homogeneous good is fixed. Hence, Home can only manipulate P^D, which it will do optimally by setting an import tariff or an export tax in the differentiated sector according to equation (46).

When there is active selection, equations (42)-(45) offer a strict generalization of the results of Haaland and Venables (2014). In line with the papers cited in Section 5.3, they assume a constant elasticity of substitution between domestic and foreign goods, $e^D = s^D_H = s^D_F \quad s^D$, that firms only differ in terms of their productivity, and that the distribution of firm-level productivity is Pareto. Crucially, they also assume that Home is small relative to Foreign in the sense that it cannot affect the number of foreign entrants, N^D_F, nor local output, Q^D_{FF}, in the differentiated sector. This implies $z_X = h^D_X = 0$ and $z_{FH} = 1/ k^D$. Under this restriction, Appendix D.3 establishes that

$$\frac{(1 + \bar{t}^D_{FH})/ (1 + \bar{t}^D_{HH})}{(1 + \bar{s}^D_{HF})/ (1 + \bar{s}^D_{HH})} = 1 + \frac{1 + e^D/ k^D}{e^D \quad 1}, \tag{48}$$

$$(1 + \bar{t}^D_{FH})/ (1 + \bar{t}^O_H) = 1 + 1/ k^D. \tag{49}$$

By equation (48), the structure of optimal trade protection within the differentiated sector is again exactly the same as in the one-sector case, with firm heterogeneity lowering trade protection if and only if there is active selection of foreign firms into exporting. [17] Furthermore, by equation (49), the same aggregate nonconvexities, $k^D < 0$, should lead

[16] If Home is an importer of the homogeneous good, $z > 1$, then Home's terms of trade unambiguously improve if both P^D and P^D_{HF} increase. Although a decrease in Home's imports of the homogeneous good imports of differentiated goods necessarily increases P^D and lowers P^D_{FH}, it only increases P^D_{HF} if Foreign's elasticity of substitution between domestic and foreign goods, e^D, is low enough. Accordingly, Home only taxes imports of the homogeneous good in this case if $e^D < z/ (r^D_{FF}(z \quad 1))$.

[17] All formulas in this section are implicitly derived under the assumption that Home and Foreign produce in both sectors. A small open economy, however, is likely to be completely specialized in only one of them. When Home is completely specialized in the differentiated sector, one can show that both equations (48) and (49) must still hold. When Home is completely specialized in the outside sector, equation (49) must again hold, but equation (48), while consistent with an optimum, is no longer necessary. Details are available upon request.

to less trade protection in the differentiated sector relative to the homogeneous sector: $(1 + \overline{t}^{D}_{FH})/(1 + \overline{t}^{O}_{H}) < 1$. This re ects the fact that given aggregate nonconvexities, the import price in the differentiated sector, P^{D}_{FH}, is a *decreasing* function of import volumes, Q^{D}_{FH}. This can again be achieved by subsidizing imports of the differentiated good, $\overline{t}^{D}_{FH} < 0$ with $\overline{t}^{O}_{H} = 0$, or by subsidizing exports of the homogeneous good, $\overline{t}^{D}_{FH} = 0$ with $\overline{t}^{O}_{H} > 0$, an expression of Lerner symmetry.

6.3 Terms-of-Trade Manipulation and Optimal Trade Policy Redux

The existing literature on optimal trade policy under monopolistic competition draws a sharp distinction between models with only intra-industry trade, like the one studied by Gros (1987), and models with both intra- and inter-industry, like the one studied by Venables (1987). In the former class of models, the standard view, as put forward by Helpman and Krugman (1989), is that terms-of-trade manipulation can be thought of as

We define terms-of-trade manipulation at the macro-level as the manipulation of the relative price of sector-level aggregate prices, not the manipulation of relative wages. In the one-sector case studied by Gros (1987), the two definitions coincide, but not otherwise. While one may view the previous distinction as semantic, this does not mean that it is either irrelevant or trivial. Part of the reason why one builds theory is to develop a common language that can be applied under seemingly different circumstances. The perspective pushed forward in this paper is that within the class of models that we consider, international trade remains another transformation activity that turns aggregate exports into aggregate imports, as summarized by the trade balance condition in (41), the shape of which determines the structure of optimal trade policy at the macro-level.

7 Concluding Remarks

In this paper, we have characterized optimal trade policy in a generalized version of the trade model with monopolistic competition and firm-level heterogeneity developed by Melitz (2003). We have organized our analysis around two polar assumptions about the set of available policy instruments. In our baseline environment, ad-valorem taxes are unrestricted so that governments are free to impose different taxes on different firms. In our extensions, ad-valorem taxes are uniform so that governments cannot discriminate between firms from the same country.

When ad-valorem taxes are unrestricted, we have shown that optimal trade policy requires micro-level policies. Specifically, a welfare-maximizing government should impose firm-level import taxes that discriminate against the most profitable foreign exporters. In contrast, export taxes that discriminate against or in favor of the most profitable domestic exporters can be dispensed with. When taxes are uniform, we have shown that the selection of heterogeneous firms into exporting tends to create aggregate nonconvexities that lowers the overall level of trade protection. Under both assumptions, we have highlighted the central role that terms-of-trade manipulation plays in determining the structure of optimal trade taxes at the macro-level, thereby offering a unifying perspective on previous results about trade policy under monopolistic competition.

We conclude by pointing out three limitations of the present analysis that could be relaxed in future research. The first one is the assumption that all firms charge a constant markup. In general, a government that manipulates its terms-of-trade may do so by imposing different taxes on different firms and incentivize them to charge different markups. In practice, we know that firms of different sizes tend to have different markups and different pass-through rates; see e.g. Berman, Martin and Mayer (2012), Goldberg, Loecker, Khandelwal and Pavcnik (2015), and Amiti, Itskhoki and Konings (2015). While this channel is not directly related to the selection of heterogeneous firms into exporting, this is another potentially important mechanism through which firm heterogeneity may affect the design of optimal trade policy.

The second limitation is that fixed exporting costs are assumed to be paid in the exporting country. This implies that all trade is trade in goods. If fixed costs were paid in the importing country, trade would also include trade in services, and manipulating the prices of such services would also be part of the objective of a welfare-maximizing government. More generally, our analysis abstracts from intermediate goods and global supply chains, which is another exciting area for future research on optimal trade policy; see Blanchard, Bown and Johnson (2015) for a first step in this direction.

The final limitation is that governments have access to a full set of tax instruments. As discussed in the previous section, when domestic instruments are restricted, trade policy would be called for not only to improve a country's terms of trade, but also to help in mitigating domestic distortions. We know little about the implications of trade models with firm heterogeneity for the design of optimal industrial policy. They may be particularly relevant in economies where credit markets are imperfect. In short, much remains to be done on the normative side of the literature.

References

Amiti, Mary, Oleg Itskhoki, and Jozef Konings, "International Shocks and Domestic

Costinot, Arnaud, Dave Donaldson, Jonathan Vogel, and Ivan Werning, "Comparative Advantage and Optimal Trade Policy," *Quarterly Journal of Economics*, 2015, *130* (2), 659–702.

_ **, Guido Lorenzoni, and Ivan Werning**, "A Theory of Capital Controls as Dynamic Terms-of-Trade Manipulation," *Journal of Political Economy*, 2014, *122* (1), 77–128.

Demidova, Svetlana, "Trade Policies, Firm Heterogeneity, and Variable Markups," *mimeo McMaster University*, 2015.

_ **and Andres Rodríguez-Clare**, "Trade policy under rm-level heterogeneity in a small economy," *Journal of International Economics*, 2009, *78* (1), 100–112.

Dhingra, Swati and John Morrow, "The Impact of Integration on Productivity and Welfare Distortions Under Monopolistic Competition," *mimeo, LSE*, 2012.

Dixit, Avinash, "Tax policy in open economies," in A. J. Auerbach and M. Feldstein, eds., *Handbook of Public Economics*, Vol. 1 of *Handbook of Public Economics*, Elsevier, 1985, chapter 6, pp. 313–374.

Dixit, Avinash K. and Joseph E. Stiglitz, "Monopolistic Competition and Optimum Product Diversity," *The American Economic Review*, 1977, *67* (3), 297–308.

Eaton, Jonathan and Samuel Kortum, "Technology, Geography and Trade," *Econometrica*, 2002, *70* (5), 1741–1779.

Everett, Hugh, "Generalized Lagrange Multiplier Methods for Solving Problems of Optimal Allocation of Resources," *Operations Research*, 1963, *11* (3), 399–417.

Feenstra, Robert C., "Measuring the Gains from Trade under Monopolistic Competition," *Canadian Journal of Economics*, 2010, pp. 1–28.

Felbermayr, Gabriel J., Benjamin Jung, and Mario Larch, "Optimal Tariffs, Retaliation and the Welfare Loss from Tariff Wars in the Melitz Model," *Journal of International Economics*, 2013, *89* (1), 13–25.

Flam, Harry and Elhanan Helpman, "Industrial Policy under Monopolistic Competition," *Journal of International Economics*, 1987, *22* (79-102).

Goldberg, Pinelopi, Jan De Loecker, Amit Khandelwal, and Nina Pavcnik, "Prices, Markups and Trade Reform," *forthcoming, Econometrica*, 2015.

Gopinath, Gita, Elhanan Helpman, and Kenneth Rogoff, eds, *Handbook of International Economics*, Vol. 4, Elsevier, 2014.

Gros, Daniel, "A Note on the Optimal Tariff, Retaliation and the Welfare Loss from Tariff Wars in a Framework with Intra-Industry Trade,"

Venables, Anthony J., "Trade and trade policy with differentiated products: A Chamberlinian-Ricardian model," *Economic Journal*, 1987,*97* (700-717).

A Proofs of Section 3

A.1 Existence and Uniqueness of the Solution to the Lagrangian Problem (Section 3.1)

Let us first rewrite equation (11) as

$$1 = N_H \left[\int_{\Phi_{Hj}} (\mu_H a_{Hj}(\varphi)/\lambda_{Hj})^{1-\sigma_H} dG_H(\varphi) \right]^{1/(1-\sigma_H)} Q_{Hj}^{1/\sigma_H}. \tag{A.1}$$

Since $\sigma_H > 1$, the integrand $(\mu_H a_{Hj}(\varphi)/\lambda_{Hj})^{1-\sigma_H}$ is increasing in λ_{Hj}. In addition, the set over which we integrate Φ_{Hj} is increasing in λ_{Hj}. Thus, the right-hand side of equation (A.1) is continuous and strictly decreasing in λ_{Hj}. One can check that it has limits equal to zero and infinity when λ_{Hj} goes to zero and infinity, respectively. By the Intermediate Value Theorem, there exists therefore a unique $\lambda_{Hj}(N_H)$ that satisfies (A.1) given N_H. Furthermore $\lambda_{Hj}(N_H)$ must be strictly increasing with limits equal to zero and infinity when N_H goes to zero and infinity, respectively.

Now let us turn to equation (12). Using our previous results, we can rewrite this expression as

$$\ell_H(\lambda_{HH}(N_H), \lambda_{HF}(N_H)) = 0, \tag{A.2}$$

with ℓ_H such that

$$\ell_H(\lambda_{HH}, \lambda_{HF}) = \sum_{j=H,F} \int \min_{\tilde{q}} \left[l_{Hj}(\tilde{q}(\varphi), \varphi) - \lambda_{Hj}(\tilde{q}(\varphi))^{1/\mu_H} \right] dG_H(\varphi) + f^e$$

satisfies (9b) for $j = H, F$, we must have

$$N\Big[\sum_{j=H,F} \int_{\Phi} l_{Hj}(q_{Hj}(\varphi), \varphi)\, dG_H(\varphi) + f^e_H\Big] \quad\quad N_H\Big[\sum_{j=H,F} \int_{\Phi} l_{Hj}(q_{Hj}(\varphi), \varphi)\, dG_H(\varphi) + f^e_H\Big].$$

Since (q_{HH}, q_{HF}, N) satisfies (9b) $j = H, F$, we must also have

$$\sum_{j=H,F} \lambda_{Hj}\Big[\int_{\Phi} N(q_{Hj}(\varphi))^{1/\mu_H} dG_H(\varphi) \quad\quad Q_{Hj}^{1/\mu_H}\Big] = \sum_{j=H,F} \lambda_{Hj}\Big[\int_{\Phi} N_H(q_{Hj}(\varphi))^{1/\mu_H} dG_H(\varphi) \quad\quad Q^{1/\mu_H}$$

B Proofs of Section 4

B.1 Lemma 4

Proof of Lemma 4First, consider the marginal rate of substitution, MRS_j U_{Hj} / U_{Fj}, in country $j = H, F$

By definition of $P_{FH}(\cdot,\cdot)$, we know that

$$P_{FH}Q_{FH} = \int_{\Phi} N_F \mu_F a_{FH}(\varphi) q_{FH}(\varphi) dG_F(\varphi).$$

Together with equation (15), this implies

$$\frac{P_{FH}}{(N_F)^{1/(1-\sigma_F)}\mu_F} = \frac{\int_{\Phi^u_{FH}} (a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi) + \int_{\Phi^c_{FH}} (\theta_{FH}(\varphi))^{\mu_F} a_{FH}(\varphi)^{1-\sigma_F} dG_F(\varphi)}{(\mu_F^2/\lambda_{FH})^{\sigma_F}(N_F)^{\sigma_F/(1-\sigma_F)}}.$$

Using equations (15) and (16), one can also check that

$$\frac{(\mu_F^2/\lambda_{FH})^{\sigma_F-1}}{N_F} = \int_{\Phi^u_{FH}} (a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi) + \int_{\Phi^c_{FH}} (\theta_{FH}(\varphi) a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi).$$

Combining the two previous expressions, we then obtain

$$\frac{P_{FH}}{(N_F)^{1/(1-\sigma_F)}\mu_F} = \frac{\int_{\Phi^u_{FH}} (a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi) + \int_{\Phi^c_{FH}} (\theta_{FH}(\varphi))^{\mu_F} a_{FH}(\varphi)^{1-\sigma_F} dG_F(\varphi)}{\int_{\Phi^u_{FH}} (\varphi))^{1-\sigma} G_F(\varphi \ldots}$$

(iv) goods prices such that

$$p_{ij}(j) = \begin{cases} \bar{p}_{ij}(j) & \text{, if } (m \quad 1) a_{ij}(j) q_{ij}(j) \quad f_{ij}(j), \\ ¥ & \text{, otherwise,} \end{cases} \qquad \text{(B.13)}$$

and

$$P_{ji}^{1 \quad s_j} = \quad {}_{F},$$

$f \quad f_{ij}¥ <$

which imply

$$L_{HF} = \left[N_H \int_{\Phi_{HF}} (\mu_H a_{HF}(\varphi))^{1-\sigma_H} dG_H(\varphi)\right]^{1/(1-\sigma_H)}. \tag{B.22}$$

By equations (B.10), (B.11), (B.13), (B.15), and (B.14), we know that

$$P_{HF}^{1-\sigma_H} = (P_{HF})^{1-\sigma_H} \int_{\Phi_{HF}} N_H [a_{HF}(\varphi)/L_{HF}]^{1-\sigma_H} dG_H(\varphi).$$

Combining this expression with equation (B.22), we obtain

$$P_{HF} = P_{HF}. \tag{B.23}$$

By equations (B.21) and (B.23), condition (B.20) must hold, which establishes that condition (1) also for goods exported by home firms.

We can use a similar logic to analyze micro-level quantities at home. Given equations (B.7), (B.11), (B.13), and (B.15), condition (1) holds for goods locally sold by home firms if

$$(\mu_H a_{HH}(\varphi))/ l_{HF}. \tag{B.23}$$

that

$$\lambda_{FH}/\mu_F = \Big[\int_{\Phi^u_{FH}} N_F \mu_F (a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi)$$
$$+ \int_{\Phi^c_{FH}} N_F \mu_F (q_{FH}(\varphi) a_{FH}(\varphi))^{1-\sigma_F} dG_F(\varphi)\Big]^{1/(1-\sigma_F)}.$$

Together with equation (B.14), this expression leads to equation (B.26). Hence, condition (1) must also hold for goods exported by foreign firms.

Third, consider the free entry condition (5). Abroad, equations (A.5) and (B.7) imply

$$P_{FF}Q_{FF}/N_F + P_{FH}Q_{FH}/N_F - \sum_{j=H,F} \int_{\Phi} l_{Fj}(q_{Fj}(\varphi),\varphi) dG_F(\varphi) = f^e_F. \tag{B.27}$$

For foreign goods that are locally sold, equations (B.8), (B.9), (B.10), (B.12), (B.13), and (B.14) imply

$$P_{FF}Q_{FF}/N_F = \int_{\Phi} \mu_F a_{FF}(\varphi) q_{FF}(\varphi) dG_F(\varphi). \tag{B.28}$$

For foreign goods that are exported, the definition of $P_{FH}(Q_{FH}, N_F)$ and equations (B.7) and (B.8) imply

$$P_{FH}Q_{FH}/N_F = \int_{\Phi} \mu_F a_{FH}(\varphi) q_{FH}(\varphi) dG_F(\varphi). \tag{B.29}$$

Equations (B.27)-(B.29) lead to the free entry condition (5) abroad. At home, equations (12) and (B.7) directly imply (5).

Fourth, consider the labor market condition (6). Abroad, this condition derives from equations (A.6), (B.10), and (B.27). At home, the resource constraint (17c) must be binding at the first-best allocation,

$$L_H(Q_{HH}, Q_{HF}) = L_H. \tag{B.30}$$

Condition (6) then derives from the definition of $L_H(Q_{HH}, Q_{HF})$ and equations (B.7), (B.8), (B.10), and (B.30).

Finally, consider condition (3). Abroad, we know from equations (A.3) and (A.6) that at the first-best allocation,

$$U_{HF}/U_{FF} = P_{HF}/P_{FF},$$
$$P_{FF}Q_{FF} + P_{FH}Q_{FH} = L_F.$$

Thus equation (B.12), (B.8), (B.19), and (B.23) imply that condition (3) holds abroad. At Home, we know from equations (18)-(20) that at the first-best allocation

$$U_{FH}/U_{HH} = (1+\tau)(L_{HF}P_{FH}/L_{HH}P_{HF}), \tag{B.31}$$

Equations (B.25) and (B.31) imply

$$U_{FH}/\ U_{HH} = (1 + \mathrm{t}\ \ )(P_{FH}/\ P_{HH}).$$

Substituting for $(1 + \mathrm{t}\ \ )P_{FH}$ using equation (B.6), we then get

$$U_{FH}/\ U_{HH} = P_{FH}/\ P_{HH}. \tag{B.32}$$

At the rst-best allocation, we also know that constraint (17b) must be binding, which implies

$$P_{FH}Q_{FH} = P_{HF}Q_{HF},$$

and in turn, using equation (B.8),

$$P_{HH}Q_{HH} + P_{FH}Q_{FH} = P_{HH}Q_{HH} + P_{HF}Q_{HF}. \tag{B.33}$$

Since conditions (1) and (4) hold for goods sold by home   Gn/F112 7.9701 Tf 6.113 -1.759 Td [(H)-90(H)]TJ/F112 which, using equations (B.13), (B.17), and (B.35), leads to

$$P_{FH}Q_{FH} = N_F \quad \mathrm{m}_{F}(1 + t_{HF}(j))a_{FH}(j)q_{FH}(j)dG_H(j). \qquad \text{(B.35)}$$

From equations (B.8), (B.10) and (B.29), we also know that

$$P_{FH}Q_{FH} = N_F \quad \mathrm{m}_{F}a_{FH}(j)q_{FH}(j)dG_F(j). \qquad \text{(B.36)}$$

Combining equation (B.18) with equations (B.34), (B.35), and (B.36), we nally obtain

NH Combndion (

$$P_{HH}Q_{HH} + P_{FH}Q_{FH} = w_H = H$$

Like in Section 3.1, the comparison of equations (1), (4), (2), and (5), on the one hand, and equations (C.1), (11), and (C.3), on the other hand, imply that the outputs of foreign varieties and the measure of foreign entrants in the decentralized equilibrium, conditional on Q_{FH} and Q_{HF}, must coincide with the solution of (30), conditional on Q_{FH} and $Q_{FF} = Q_{FF}(Q_{FH})$. Since the outputs of foreign varieties and the measure of foreign entrants satisfy (6), we must therefore have

$$L_F(Q_{FH}, Q_{FF}(Q_{FH})) = L_F,$$

which establishes equation (33). To conclude, note that by the Envelope Theorem, we must have

$$\partial L_F(Q_{FH}, Q_{FF})/\partial Q_{Fj} = \lambda_{Fj} Q_{Fj}^{1/\sigma_F}/\mu_F. \tag{C.4}$$

Conditional on Q_{FH} and $Q_{FF} = Q_{FF}(Q_{FH})$, equations (A.4) and (C.2) further imply that

$$\lambda_{FF} = P_{FF}(Q_{HF}, Q_{FH})(Q_{FF}(Q_{FH}))^{1/\sigma_F}. \tag{C.5}$$

Similarly, equations (28) and (C.2) imply that

$$\lambda_{FH} = P_{FH}(Q_{FH}, N_F(Q_{FH}))Q_{FH}^{1/\sigma_F}. \tag{C.6}$$

Equation (32) follows from equations (C.4)-(C.6). □

C.2 Marginal Rate of Transformation is Homogeneous of Degree Zero (Section 5.2)

C.4 Lemma 7

Proof of Lemma 7. In Section C.2, we have established that

$$MRT_F(Q_{FH}, Q_{FF}) = \frac{A_{FH}(M_F(Q_{FH}, Q_{FF})/ Q_{FH})}{A_{FF}(M_F(Q_{FH}, Q_{FF})/ Q_{FF})}.$$

with M_F, A_{FH}, and A_{FF} implicitly determined by equations (C.7)-(C.10). Taking log and totally differentiating the previous expression with respect to Q_{FH}, we get

$$\frac{d \ln MRT_F(Q_{FH}, Q_{FF}(Q_{FH}))}{d \ln Q_{FH}} = e^{A}_{FH}(\quad (1 \quad e^{M}_{FH}) + e^{Q}_{F} e^{M}_{FF}) + e^{A}_{FF}(\quad e^{M}_{FH} + e^{Q}_{F}(1 \quad e^{M}_{FF}))),$$

and, in turn,

$$e^{A}_{FF} \quad e^{F}_{FF} = \Big(\frac{1 \quad (s_F \quad 1)e_{FF}}{s_F}\Big)\Big(1 + \frac{(s_F \quad 1) \quad {}_{j_{FF}}(j \,/\, j_{FF})^{s_F \quad 1} dG_F(j\ )}{1 \quad G_F(j_{FF})}\Big).$$

Hence, the suf cient condition (C.21) can be rearranged as

$$\frac{1}{s_F \quad 1}\frac{j_{FF}\, g_F(j_{FF})}{1 \quad G_F(j_{FF})} > (s_F \quad 1) \quad {}_{j_{FF}}(j \,/\, j_{FF}(j$$

with $MRS_H^{HO}\quad (\partial U_H / \partial Q^D_{HH}) / (\partial U_H / \partial U^O_H)$ the marginal rate of substitution for Home between Home's differentiated good and the homogeneous good and $L^D_{HH}\quad \partial L^D_H / \partial Q_{HH}$ the marginal cost of aggregate output for the local market at home. Like in Section 4.3, one can use the Envelope Theorem to show that

$$L^D_{HH} = ({}_{\Omega^D_{HH}} N^D_H (a_{HH}(j))^{1\ \sigma^D_H} dG^D_H(j))^{1/(1\ \sigma^D_H)}. \tag{D.3}$$

In the decentralized equilibrium with taxes, utility maximization at home implies

$$MRS_H^{HO} = P^D_{HH}, \tag{D.4}$$

with the aggregate price index such that

$$P^D_{HH} = ({}_{\Omega^D_{HH}} N^D_H ((1 + \bar{t}^D_{HH})\mu^D_H a_{HH}(j)/(1 + \bar{s}^D_{HH}))^{1\ \sigma^D_H} dG_H(j))^{1/(1\ \sigma^D_H)}. \tag{D.5}$$

In the decentralized equilibrium abroad, we know that

$$Q^D_{FF} = (\beta_F L_F + X^O_H)/P^D_{FF} - (P^D_{FH}/P^D_{FF})Q^D_{FH}$$

with price indices such that

$$P^D_{FF} = \left(\int_{\Phi_{FF}} N^D_F (\mu^D_F a_{FF}(\varphi))^{1-\sigma^D_F} dG_F(\varphi)\right)^{1/(1-\sigma^D_F)},$$

$$P^D_{FH} = \left(\int_{\Phi_{FH}} N^D_F (\mu^D_F a_{FH}(\varphi))^{1-\sigma^D_F} dG_F(\varphi)\right)^{1/(1-\sigma^D_F)},$$

$$N^D_F = \frac{\beta_F L_F + X^O_H}{(\sigma_F - 1)[f^e_F + \sum_{j=H,F} \int_{\Phi_{Fj}} f_{Fj}(\varphi) dG_F(\varphi)]}.$$

In the absence of active selection, we can treat Φ_{FF} and Φ_{FH} as fixed. Thus, the previous equations imply

$$d \ln Q^D_{FF} / d \ln X^O_H = (\mu^D_F X^O_H)/(P_{FF} Q_{FF}).$$

Combining this expression with equation (D.9), we obtain

$$\eta^D_X = (\mu^D_F X^O_H)/(\varepsilon^D P_{FF} Q_{FF}). \tag{D.10}$$

Finally, consider $\zeta_{FH} \equiv \partial \ln P^D_{FH}/\partial \ln Q^D_{FH}$ and $\zeta_X \equiv \partial \ln P^D_{FH}/\partial \ln X^O_H$. In the absence of active selection, we must have

$$\zeta_{FH} = 0, \tag{D.11}$$

$$\zeta_X = \frac{1}{1-\sigma^D_F} \frac{X^O_H}{(P_{FF} Q_{FF} + P_{FH} Q_{FH})}. \tag{D.12}$$

Combining equations (42) and (43) with equations (D.6), (D.8), (D.10), (D.11), and (D.12), we obtain

$$\tau^D = \frac{1}{(\varepsilon^D - 1) x^D_{FF}},$$

$$\tau^O = \frac{(1 - r^D_{FF})(z/r^D_{FF} + (1-z)\varepsilon^D)}{\varepsilon^D(\sigma^D_F - 1) + (1 - r^D_{FF})(\sigma^D_F z/r^D_{FF} + (1-z)\varepsilon^D)},$$

where the first expression uses the fact foreign expenditure and revenue shares are related through $(1/x^D_{FF} - 1) = (1/r^D_{FF} - 1)z$. Equations (46) and (47) derive from equations (44) and (45) and the two previous expressions.

economy, then $z_X = h^D_X = 0$ and $z_{FH} = 1/k^D$. In addition, setting $r^D_{FF} = 1$ in equation (D.7), we obtain $h^D_{FH} = \quad 1/k^D$. The last elasticity, h^D_{HF}, is unaffected by the fact that Home is a small open economy: $h^D_{HF} = \quad 1/e^D$ by equation (D.6). Combining the previous observations with equations (42) and (43), we get

$$t^D = (1 + e^D/k^D)/(e^D \quad 1),$$
$$t^O = 1/k^D.$$

Equations (48) and (49) derive from equations (44) and (45) and the two previous expressions.