

# Gravity with Granularity

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## Abstract

We evaluate the consequences of oligopolistic behavior for the estimation of gravity equations for trade flows. With oligopolistic competition, firm-level gravity equations based on a standard CES demand framework need to be augmented by markup terms that are functions of firms' market shares. At the aggregate level, the additional term takes the form of the exporting country's market share in the destination country multiplied by an exporter-destination-specific Herfindahl-Hirschman index. For both cases, we show how to construct appropriate correction terms that can be used to avoid problems of omitted variable bias. We demonstrate the quantitative importance of our results for combined French and Chinese firm-level export data as well as for a sample of product-level imports by European countries. Our results show that correcting for oligopoly bias can lead to substantial changes in the coefficients on standard gravity regressors.

*Keywords:* Gravity Equation, Oligopoly, CES Demand, Aggregative Game

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# 1 Introduction

Gravity equations have been the predominant tool for analyzing the determinants of bilateral trade flows since their introduction by Tinbergen (1962) over 50 years ago. In their most basic form, gravity equations predict that trade between countries is a log-linear function of the economic mass of the two trading partners and bilateral frictions such as distance or tariffs. Even in this simple form, gravity equations have substantial explanatory power, often explaining in excess of 70-80% of the variation in the trade flows between countries. Starting with Anderson (1979), researchers have shown that gravity equations can be derived from a number of mainstream theoretical frameworks, allowing a tight link to economic welfare analysis. Not surprisingly then, gravity equations have become the workhorse tool for evaluating trade-related economic policies, such as trade agreements, WTO membership or currency unions.





which is our key contribution.

Finally, in work concurrent to and independent of ours, Heid and Staehler (2020) propose an extension of Arkolakis, Costinot, and Rodriguez-Clare (2012)'s formula to evaluate the gains from trade under oligopoly. To consistently estimate parameters necessary for the quantification of their model, they derive and estimate an aggregate gravity equation in oligopoly under the assumption that all industries are symmetric and each country hosts one firm per industry. Moreover, they have to take key parameters (such as price elasticities) from the existing literature, although the underlying estimation procedures are inconsistent with oligopolistic competition. By contrast, the firm- and industry-level gravity equations that we derive and estimate allow industries to differ in an arbitrary way and each country to host multiple (heterogeneous) firms. Moreover, we propose an adaptation of existing estimation procedures to obtain key parameter estimates in a way consistent with oligopolistic behavior.

The rest of this paper is organized as follows. In Section 2, we derive a firm-level gravity equation from a CES-demand framework with oligopolistic quantity competition. We also discuss how to deal with selection and heteroscedasticity in estimating our oligopolistic firm-level gravity equation. Next, we show in Section 3 how to modify the Feenstra-Broda-Weinstein estimation procedure to account for oligopolistic behavior and obtain demand and supply elasticity estimates. In Section 4, we derive our correction term for aggregate product-level trade flows. We also discuss how to adapt the methodology developed by Helpman, Melitz, and Rubinstein (2008) so as to deal with selection in the estimation of sector-level gravity under oligopoly. In Section 5, we describe the data sources and present the empirical results from our firm- and sector-level gravity estimations. In Section 6, we provide Monte Carlo simulations to evaluate the performance of our oligopoly correction term for sector-level regressions and that of our methods to deal with heteroscedasticity and selection. Finally, we conclude in Section 7. Appendix A collects proofs of our theoretical results. Results obtained when assuming price instead of quantity competition are presented in Appendix B. Appendix C contains lists of the countries present in our datasets.

## 2 Firm-Level Gravity in Oligopoly

We consider a multi-country world with a continuum of sectors, indexed by  $z$ . The representative consumer in country  $n$  maximizes

$$U_n = \int_{z \in Z} \int_{j \in J_n(z)} a_{jn}^{1-\sigma(z)} q_{jn}^{\sigma(z)-1} A_n^{\frac{1}{\sigma(z)-1}} dz;$$

where  $a_{jn}(z)$

where  $c_{in}$  is a firm-destination-specific cost shifter and  $\tau_{in}$  a firm-destination-specific trade cost that takes the usual iceberg form.<sup>5</sup>

We assume throughout that the returns-to-scale parameters satisfies  $\sigma > 1$ , which means that the marginal cost of production should not decrease too fast with output. This (weak) assumption guarantees that all the profit functions we consider will be unimodal.

Unlike in monopolistically competitive markets, firms take into account the impact of their actions on the CES-composite,  $Q_n$ , when setting quantities. For what follows, it is useful to generalize further the degree of strategic interaction between firms by introducing a conduct parameter,  $\theta$  (see Bresnahan, 1989): When firm  $i$  increases its output  $q_{in}$  by an infinitesimal amount, it perceives the induced effect on  $Q_n$  to be equal to  $\frac{\partial Q_n}{\partial q_{in}} = \theta \frac{Q_n}{q_{in}}$ . Under monopolistic competition, the conduct parameter takes the value of zero, whereas it is equal to one under Cournot competition. The first-order condition of profit maximization of firm  $i$  in destination  $n$  is given by

$$0 = \frac{\partial \pi_{in}}{\partial q_{in}} = \frac{p_{in} E_n a_{in}^{\frac{1}{\sigma}} q_{in}^{\frac{1}{\sigma}}}{Q_n^{\frac{1}{\sigma} + 1}} - \frac{1}{\theta} \frac{\partial Q_n}{\partial q_{in}} \frac{p_{in} E_n a_{in}^{\frac{1}{\sigma}} q_{in}^{\frac{1}{\sigma}}}{Q_n^{\frac{1}{\sigma} + 1}} - c_{in}^0(q_{in}) \quad (2)$$

is the index of product

where

$$s_{in} = \frac{a_{in}^{\frac{1}{\sigma}} q_{in}^{\frac{1}{\sigma}}}{\sum_{j \in J_n} a_{jn}^{\frac{1}{\sigma}} q_{jn}^{\frac{1}{\sigma}}} \quad (3)$$

is the market share of firm  $i$  in destination  $n$ .

Rearranging terms in equation (2) yields firm  $i$ 's optimal markup in destination  $n$ :

$$\mu_{in} = \frac{1}{\sigma} + \frac{1}{\theta} s_{in} \quad (4)$$

where  $\mu_{in} = (p_{in} - c_{in}^0(q_{in})) / p_{in}$  is the Lerner index of product

the value of its sales in market  $n$  can be written as:

$$r_{in} = p_{in} q_{in} = \frac{c_{in}}{1 + \frac{1}{\sigma}} a_{in} P_n^{1 + \frac{1}{\sigma}} E_n^{\frac{1}{1 + \frac{1}{\sigma}}} \quad (5)$$

So far, we have not imposed any structure on trade costs,  $c_{in}$  or the taste and cost shock terms,  $a_{in}$  and  $c_{in}$ . For comparison with the existing literature and to facilitate the exposition of our identifying assumptions, we now assume that the two shock terms can be decomposed log-linearly as  $\log a_{in} = \alpha_i + \alpha_n + \alpha_{in}$  and  $\log c_{in} = \gamma_i + \gamma_n + \gamma_{in}$ , respectively. We further assume that trade costs can be decomposed as  $\log c_{in} = X_{in} + \eta_{in}$  where the  $X_{in}$  include variables with bilateral variation such as (log) distance, common language or dummies for the presence of trade agreements or currency unions. Obtaining consistent estimates of the coefficients on these bilateral terms ( ) is a key objective of much of gravity equation-based research.<sup>6</sup> Finally, we again assume a three-way decomposition of the trade cost error term,  $\eta_{in} = \eta_i + \eta_n + \eta_{in}$ .

Taking the logarithm of equation (5) yields a firm-level gravity equation of the form

$$\log r_{in} = \alpha_n + \alpha_i + \frac{1}{1 + \frac{1}{\sigma}} X_{in} + \frac{1}{1 + \frac{1}{\sigma}} \log(1 + \eta_{in}) + \eta_{in} \quad (6)$$

where  $\alpha_n$  and  $\alpha_i$  summarize destination- and firm-specific terms and

$$\eta_{in} = \frac{1}{1 + \frac{1}{\sigma}} \ln(1 + \eta_{in}^a) + (1 + \frac{1}{\sigma}) \eta_{in}^c + (1 + \frac{1}{\sigma}) \eta_{in}^i :$$

Note that under the assumption of monopolistic competition, the markup term involving  $\eta_{in}^a$  would be constant and could be subsumed in  $\alpha_i$ . In that case, estimation of (6) would yield consistent estimates of the coefficient on  $X_{in}$  provided that we control for firm and destination fixed effects ( $\alpha_i$  and  $\alpha_n$ ) and that the usual orthogonality assumptions (explicitly or implicitly) made in the gravity literature hold.<sup>7</sup>

In the presence of strategic interaction between firms, however, the markup term will

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<sup>6</sup>See, for example, Baier and Bergstrand (2007) and Rose (2000) on the effects of free trade agreements and currency unions, respectively, on trade flows.



depend on firms' market shares and will thus be correlated with the regressors of interest,  $X_{in}$ ; not including this term will lead to an omitted variable bias. For example, we would expect firms to have lower market shares in more distant markets, ceteris paribus, and hence to charge lower markups there. This implies that  $\log(1 - b_{in})$  will be higher in such markets, leading to a positive correlation between distance and the omitted variable.

Note that this problem is qualitatively different from those arising from other hard-to-observe gravity components such as expenditure ( $E_n$ ), price indices ( $P_n$ ) or firm-level marginal costs because these components can be controlled for by firm or destination fixed effects. By contrast, markups vary at the firm-destination level and the inclusion of bilateral fixed effects would make it impossible to identify separately the effect of key regressors of interest such as distance, tariffs or dummy variables for trade agreement.<sup>8</sup>

Instead, we propose to solve the omitted variable problem by constructing a proxy for the markup term in (6). Specifically, if we had estimates for  $\alpha$  and  $\beta$  and data for  $s_{in}$ , we could compute

$$b_{in} = \frac{1}{b} + \frac{b-1}{b} s_{in}$$

and estimate

$$\log e_{in} - \log r_{in} - \frac{b-1}{1+bb} \log(1 - b_{in}) = \alpha_n + \beta_i + \frac{1}{1+bb} X_{in} + \mu_{in} \quad (7)$$

Given our earlier orthogonality assumptions, using  $\log e_{in}$  instead of  $\log r_{in}$  as the dependent variable would yield a consistent estimate of  $\frac{1}{1+bb}$ . Using our estimates for  $\alpha$  and  $\beta$  would then allow recovering the parameter of interest,  $b$ .<sup>9</sup> This approach raises the question of how to estimate  $\alpha$  and  $\beta$ . In the next section, we show how to adapt the estimation procedure by Feenstra (1994) and Broda and Weinstein (2006) to our setting with firm-level data and oligopolistic competition.

## 2.1 Estimation Challenges for Firm-level Gravity

Recall that our aim is to obtain consistent estimates of the coefficients on bilateral variables using either firm or sector level data. Above we showed that after subtracting a markup correction term from firm export values, we could estimate a standard gravity equation with a set of firm-product-year and destination-product-year fixed effects as well as the bilateral

<sup>8</sup>Having a time dimension in the data would not help either because markups would then vary by firm, destination and time.

<sup>9</sup>Note the parallel to the literature on trade and quality which uses a similar approach to correct export values or quantities (e.g., Khandelwal, Schott, and Wei, 2013).

variables of interest.<sup>10</sup>

A first issue that arises is how to control for destination-specific fixed effects in a setting with firm-level export data. If we only have data for exports from a single country, it is immediately clear that we can no longer separate the impact of bilateral variables from the fixed effects. For example, if we use information on the exports of French firms only, standard bilateral variables such as common language become destination-specific as France is the only origin country in our data. Intuitively, we will not be able to distinguish whether firms' exports to a given destination are high because France and the country in question share a common language or because of other destination-specific factors such as a high price index or expenditure level. In order to address this issue, we follow Bas, Mayer, and Thoenig (2017) by combining two datasets on the exports of French and Chinese firms, respectively. This ensures that there is within-destination variation in the bilateral regressors of interest, enabling the use of destination fixed effects.

Secondly, we have so far ignored selection issues. In practice, most firms only export to a small subset of possible destinations for any given product. When estimating (7) in log-linear form, firm-product-destination observations with zero trade flows drop out. In the presence of export fixed cost  $f_{on} > 0$  there is selection into exporting in our model: firms will be more likely to export positive amounts to a given destination if they experience a positive taste, production or trade cost shock for that destination, potentially creating a non-zero correlation with the regressors of interest. For example, firms selling in more distant foreign markets will be more likely to have received a positive shock, allowing them to operate in this more difficult environment. As consequence,

$$E(\epsilon_{in}^c | X_{in}; r_{in} > 0) \neq 0; \quad E(\epsilon_{in}^a | X_{in}; r_{in} > 0) \neq 0$$

Here, we adapt an approach proposed by Bas, Mayer, and Thoenig (2017) and restrict our estimation sample to the largest three French and Chinese firm in each product category as measured by **overall** product-specific exports. The basic idea is that these firms have high overall exports because they are very productive, produce high-quality products in general (high  $\alpha_i^a$  or  $\alpha_i^c$ ) or have access to low-cost market access technologies (low  $\tau_i$ ). Such firms will tend to serve all or at least most available markets, making the destination-specific shocks less important for market entry decisions. We acknowledge that this is an imperfect solution

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<sup>10</sup>Recall that we dropped the sector/product index ( $z$ ) for most of our derivations and also ignored the time dimension to ease exposition. But these dimensions are of course present in our data, and hence price indices and expenditure levels will vary by destination, product and year, requiring the use of fixed effects at that level.

but simulation evidence by Bas, Mayer, and Thoenig (2017) shows that focusing on top exporters does indeed substantially reduce selection bias

Third, in the presence of heteroscedasticity, the log-linear gravity equation provides inconsistent coefficient estimates (Santos Silva and Tenreyro, 2006). In particular, we seek a consistent estimate of  $E(\mathbf{r}_{in}|\mathbf{X}_{in})$ . Recall that  $\mathbf{r}_{in} = \exp(\alpha_n + \beta_i + \frac{1}{1+\sigma} \mathbf{X}_{in}) \exp(\epsilon_{in})$ . Suppose that  $\text{Var}(\exp(\epsilon_{in}))$  depends on  $\mathbf{X}_{in}$ . Then  $E(\epsilon_{in})$  is a function of  $\mathbf{X}_{in}$  and thus the error term is correlated with the control variables. A solution to this problem is to include zeros in our left-hand side variable and estimate (7) in multiplicative form:

$$E(\mathbf{r}_{in}|\mathbf{X}_{in}) = \exp(\alpha_n + \beta_i + \frac{1}{1+\sigma} \mathbf{X}_{in})$$

Recent computational advances in PPML estimation (e.g., Correia, Guimaraes, and Zylkin, 2019) make it possible to include the large number of fixed effects required in our setting.<sup>11</sup>

### 3 Estimation of Supply and Demand Elasticities

Feenstra (1994) and Broda and Weinstein (2006) propose estimators for the elasticity of substitution,  $\sigma$ , based on the key identifying assumption that shocks over time to import demand and export supply for a given product are uncorrelated. The equivalent condition in our context is that  $E(\epsilon_{in}^a \epsilon_{in}^c) = 0$  for all  $i; i^0$  and  $n; n^0$ , where  $\epsilon_{in}^c = \epsilon_{in} + \epsilon_{in}^c$ . That is, we assume that the firm-destination-level elements of taste and cost shocks are uncorrelated across firms and markets.

Note that this assumption is consistent with non-zero correlations between overall taste and cost shocks (i.e.,  $E(\mathbf{a}_{in} \mathbf{c}_{in}) \neq 0$  is allowed). In particular, our method allows for a positive correlation between firm-level costs and quality ( $\epsilon_i^a$  and  $\epsilon_i^c$ ) which is to be expected if the production costs of firms producing high-quality products are higher. Likewise, our results are robust to a positive correlation between destination market quality and cost shocks ( $\epsilon_n^a$  and  $\epsilon_n^c$ ). For example, such a correlation could arise if firms sell higher-quality goods to high-income markets and incur positive costs of doing so.

We start our derivation by expressing firm-level revenues of firm  $i$  in market  $n$  in terms of expenditure shares. From equation (1),

$$\log s_{in} = \log \frac{p_{in} q_{in}}{E_n} = \log \mathbf{a}_{in} + (1 - \sigma) p_{in} + (\sigma - 1) \log \mathbf{P}_n;$$

<sup>11</sup>We include zero trade flows when estimating (7) on a product-by-product basis in Section 5.3

Now assume that we observe another firm  $i^0$  selling to the same market  $n$ . We can then subtract the logged market share of that firm to eliminate the price index:<sup>12</sup>

$$\log s_{in} - \log s_{i^0n} = \log a_{in} - \log a_{i^0n} + (1 - \alpha) (\log p_{in} - \log p_{i^0n})$$

If we observe the same two firms in another destination  $n^0$ , we can compute a double difference across the two markets as

$${}^{d,f} \log s_{in} = (1 - \alpha) {}^{d,f} \log p_{in} + {}^{d,f} \log a_{in};$$

where  ${}^f$  and  ${}^d$  denote log differences across firms and destinations, respectively. Note that double differencing only leaves the firm-destination-specific parts of the taste shocks:

$${}^{d,f} \log a_{in} = (\alpha^a$$

market shares:

$$d \log(c_{in}) = (1 + \epsilon) \log p_{in} + \log(1 - s_{in}) - \log s_{in}$$

and

$$d \log a_{in} = \log s_{in} - (1 + \epsilon) \log p_{in}$$

The sample analogue of our moment condition is then given by

$$(\beta; \gamma) = \frac{1}{J_{nn^0}} \sum_{j \in J_{nn^0}} d \log a_{in} - d \log(c_{in});$$

where  $J_{nn^0}$  denotes the set of firms active in the same two markets. Notice that we obtain one moment condition per country pair. Stacking these up allows to implement a standard GMM estimator of  $\beta$  and  $\gamma$ .<sup>13</sup>

Finally, this still leaves us with a potential selection problem in our GMM estimation procedure for  $\beta$  and  $\gamma$ . As a solution, we focus again on the top 3 Chinese and French exporters (in terms of their overall exports) for any given 6-digit HS product. Finally, in order to obtain a sufficiently large number of observations for the computation of moments in our GMM estimation, we restrict the estimates of  $\beta$  and  $\gamma$  to be identical within 2-digit HS products.

## 4 Sector-Level Gravity in Oligopoly

In this section, we study sector-level trade flows in the oligopoly model of Section 2. We first analyze the equilibrium in a given market using an aggregative games approach (Nocke and Schutz, 2018b; Anderson, Erkal, and Piccinin, 2020). We then leverage Nocke and Schutz (2018a)'s approximation techniques to derive a sector-level gravity equation that accounts for oligopolistic behavior.

**Oligopoly analysis in a given destination market.** Consider sector  $z$  in destination  $n$ . Dropping reference to both  $z$  and  $n$  to ease notation, we define the market-level aggregator  $H$  as

$$H = Q^{-1} = \sum_{j \in J} a_j^{-1} q_j^{-1}$$

<sup>13</sup>In practice, this means that we need to observe a sufficiently large number of firms selling in the same sector in at least three different markets.

and firm  $i$ 's type  $T_i$  as

$$T_i = a_i \frac{E}{c_i} \frac{1}{1 + \frac{1}{\sigma_i}} \quad (8)$$

Plugging these definitions into equation (2), making use of equation (3), and rearranging, we obtain:

$$1 - s_i = s_i \frac{1 + \frac{1}{\sigma_i}}{T_i} \frac{H}{T_i} \quad (9)$$

where  $\sigma_i$  is the conduct parameter introduced in Section 2. As the left-hand side is strictly decreasing in  $s_i$  and the right-hand side is strictly increasing in  $s_i$ , the equation has a unique solution in  $s_i$ , which we denote  $S(T_i=H; \sigma_i)$  | the **market-share fitting-in function**. It can easily

We are interested in these aggregate exports when firms compete in a Cournot fashion, i.e., when  $\eta = 1$ . Unfortunately, there is no closed-form solution to  $s_o(1)$ . Our approach therefore entails approximating  $s_o(1)$  at the first order.

As we show in the following, the approximation relies on two versions of the Herfindahl-Hirschman index (HHI), namely the HHI of all firms selling in the destination market  $n$ ,

$$HHI_n(\theta) = \sum_{j \in J} s_{jn}(\theta)^2;$$

and the (normalized) HHI of all those exporters in country  $o$  that sell in the destination market  $n$ ,

$$HHI_{on}(\theta) = \sum_{j \in E} \frac{s_{jn}(\theta)^2}{s_{on}(\theta)};$$

We obtain:

**Proposition 2.** At the first order, in the neighborhood of  $\theta = 0$  (monopolistic competition conduct), the logged joint market share in destination of the firms from export country  $o$  is given by

$$\log s_{on}(\theta) = \log s_{on}(0) + \frac{1}{1+\eta} \left[ HHI_n(\theta) s_{on}(\theta) + HHI_{on}(\theta) \right] \theta + o(\theta^2)$$

where the second line follows from the approximation in equation (11).

## 4.1 Estimation Challenges for Sector-level Gravity

In order to consistently estimate gravity equations at the sector-level, we need to correct for selection of high-quality, low-cost firms into high-trade-cost destinations. For estimating firm-level gravity, we followed Bas, Mayer, and Thoenig (2017) in focusing on the top 3 exporters from the origin country. This is, of course, not possible when using sector-level data. Here, we instead adapt the methodology developed by Helpman, Melitz, and Rubinstein (2008) (from now on HMR) for gravity estimation with heterogeneous firms and constant markups to oligopoly. To avoid multiplicity of equilibria, we assume that, at the entry stage, firms behave as under monopolistic competition and thus take the aggregate price index as given.

Under this hypothesis, firm  $i$  from origin  $o$  enters market  $n$  as long as

$$r_{in} = \frac{r_{in}}{f_{on}}$$

where

$$r_{in} = \frac{c_{in}}{1} \left( \frac{1}{1+\mu} \right)^{\frac{1}{1+\mu}} \left( \frac{1}{i} \right)^{\frac{1}{1+\mu}} P_n^{-1} E_n^{\frac{1+\mu}{1+\mu}}$$

and  $\tau_{ij}$

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Then, we obtain the empirical specification for the extensive-margin decision with sector-level export data as the following Probit model:

$$y_{on} = (\alpha_0 + \alpha_n + \mathbf{X}_{on});$$

where  $\alpha_0$  is an intercept term and  $\alpha_n$  is a sector fixed effect.

We can thus consistently estimate the intensive-margin trade elasticity  $\frac{1}{1+\mu}$ .

## 5 Empirical Implementation

In this section, we show how to implement our methods for firm- and sector-level gravity estimations empirically. We first discuss our datasets. We then present basic descriptive statistics on our data and the GMM estimates for  $\mu$  and  $\sigma$ . Finally, we run firm and sector-level regressions with and without oligopoly correction terms and investigate if and under which circumstances ignoring oligopolistic behavior can lead to quantitatively important coefficient bias.

### 5.1 Data Sources

As discussed, we use annual firm-level export data for French and Chinese exporters provided by the two countries' customs authorities for the years 2000-2010. In each dataset, we observe all the products and destinations to which a firm exports, as well as the quantity and value of the underlying flow. Both datasets record export data at the 8-digit level but we aggregate this information up to the 6-digit level of the Harmonised System (HS) which is the lowest level at which the two national classifications are comparable with each other. Because we observe both values and quantities, we can compute unit values which are a commonly used proxy for prices in the trade literature.

A final challenge for our firm-level analysis is to obtain information on market shares at a level of disaggregation that is sufficient to capture meaningful strategic interaction between firms. To our knowledge, the only suitable database here is Eurostat's PRODCOM database which allows computation of absorption at a level at, or close to, HS 6-digit.<sup>14</sup> Together

using PRODCOM is that absorption data is only available for approximately thirty European countries. After combining our data sources, we end up with information on export values and quantities as well as market shares for 32 European destination markets, approximately 1,800 products and 250,000 exporters for the period 2000-2010.<sup>15</sup>

For our sector-level gravity regressions, we require product-level data on the value of bilateral exports, absorption data for the computation of market shares and exporter-destination-product-specific HHIs. To make the estimation sample consistent with our firm-level regressions, we aggregate our firm-level data at the 6-digit HS level and use the firm level data to compute exporter HHIs.<sup>16</sup> Finally, we source information on bilateral distance from CEPII.

## 5.2 Descriptive Statistics

The key determinants of our oligopoly correction term are firm-level market shares as well as estimates for demand elasticities ( $\epsilon$ ) and returns to scale ( $\rho$ ). The first line of Table 1

firm enjoys a market share of almost 30%. In the third column we show the market shares for the sample of the top 3 exporters (i.e., the largest three French and Chinese firms in terms of total export values for a given 6-digit product and year). The mean market share in this sample is around 3.9% and at the 95th percentile it equals around 18%. In the final column, we present the cumulative market shares of the top-3 exporters. For them, the average cumulative market share is equal to 7.30% and at the 95th percentile, they have a total market share of 33.4%. Thus, there is a small set of exporters with large market shares in most markets.

Table 1: Summary Statistics for French and Chinese Firm-Level Market Shares

Table 2: Summary Statistics for Sector-Level Market Shares and Exporter HHIs

	Exporter Herfindahl	Destination Market Share	Markup Correction Term
Mean	0.55	9%	0.18
5th pctile	0.08	0.01%	0.0004
10th pctile	0.13	0.06%	0.001
Median	0.50	2%	0.03
90th pctile	1	24%	0.41
95th pctile	1	42%	0.82

Note: data for 6-digit HS sectors. Sample 2000-2010. Markup correction term computed for  $\alpha = 5$ ,  $\beta = 0$ .

numbers are very similar to estimates at comparable levels of aggregation estimated in the literature (e.g., Broda and Weinstein, 2006).

Table 3: Price Elasticities and Returns-to-Scale Estimates { Cournot Competition

Mean	5.39	0.34
25th Percentile	2.22	0.03
Median	3.74	0.10
75th Percentile	7.50	0.30
Min	1.01	-0.13
Max	26.07	4.46
Standard Deviation	4.07	0.69
HS 2-digit products	78	78

Note: Table shows descriptive statistics for estimates of  $\alpha$  and  $\beta$ . Estimates computed using 6-digit HS firm-level information but constrained to be identical within 2-digit HS products.

### 5.3 Firm-Level Gravity Estimation Results

We now turn to the estimation of our firm-level gravity equations with and without correction for oligopoly bias. In all firm-level regressions, we consider the top-3 exporters of any given 6-digit product as potential exporters of that product to any given destination and  $\mathbb{1}$  in the zero export flows if they do not export the product to a destination. As a first step, we pool across all firms in our data and estimate equations 6 and 7 via PPML using a full set of firm-product-year and destination-product-year fixed effects. We aim at identifying the intensive-margin trade elasticity given by  $\frac{1}{\alpha}$



Table 6: Firm-Level Gravity Estimates,  $\alpha = 5:39$  and  $\beta = 0:34$ .

Regressor	PPML w/ corr	PPML w/o corr	OLS w/ corr	OLS w/o corr
log distance	-1.201*** (0.118)	-0.874*** (0.0210)	-0.248*** (0.0142)	-0.231*** (0.0136)
$\hat{\alpha}_{distance}$	0.792	0.577	0.160	0.149
Observations	11,955,786	11,955,786	708,392	708,386
R-squared			0.05	0.06
Firm-year FE	YES	YES	YES	YES
Product-dest.-year FE	YES	YES	YES	YES

Note: Firm-level data, pooled across sectors. Results for top 3 exporters. Cournot model with mean of estimated  $\alpha$  and  $\beta$ . Standard errors in brackets, clustered at the destination-year level.

reduce their markups in markets where they face higher variable trade costs and thus have lower market shares. As a consequence, firm-level export values decrease by less than they would have decreased under constant markups. The point estimate on distance corresponds to  $(\alpha - 1) \hat{\alpha}_{distance}$ , where  $\hat{\alpha}_{distance}$  is the fundamental trade cost elasticity of distance. Given values of  $\alpha$  equal to 5 and  $\beta$  equal to zero, the implied values for this coefficient are 0.354 in column (1) and 0.199 in column (2), implying a downward bias in coefficient magnitude of around 44 percent. Note that while distance is not a policy variable, a very similar attenuation bias would arise for any iceberg-type variable, such as ad-valorem tariffs or transport costs.

In columns (3) and (4) we report estimates for the log-linear versions of equations (7) and







the firm level (around 10%).

Table 8: Sector-level Gravity Estimates without Controlling for Selection

Regressor	PPML w/o corr	PPML w/ corr	OLS w/o corr	OLS w/ corr
log distance	-0.263 (0.168)	-0.186 (0.147)	-1.128*** (0.195)	-1.260*** (0.216)
Observations	107,064	107,064	66,563	66,563
R-squared			0.314	0.285
Product-origin FE	YES	YES	YES	YES
Product-dest. FE	YES	YES	YES	YES

Note: Sector-level data. Cournot model with  $\alpha = 5$  and  $\beta = 0$ . Standard errors clustered at destination level

We now apply the HMR methodology in order to correct for selection into exporting in addition to the markup bias and to obtain correct estimates of the intensive-margin trade elasticity. To apply the HMR method, we first estimate the propensity to export (extensive margin) using a Probit estimator. We include 2-digit-product-origin and 2-digit-product-destination fixed effects.<sup>19</sup> To proxy for fixed market entry costs between origin  $o$  and destination  $n$ , we also follow HMR: we add dummies for the business startup time and the startup cost being above the sample median for both the origin and the destination country. By assumption, these variables only impact on the fixed export cost but not on the iceberg-type trade costs and thus exclusively affect firms' entry decision but not their quantity choice con-

Table 9: Sector-level Gravity Estimates { Export Propensity

Regressor	Export > 0
log distance	-0.420*
high startup cost	-1.340***
long startup time	-2.102***
Observations	107,064
Product-origin FE	YES
Product-dest. FE	YES

Note: Sector-level data. HMR first-stage Probit regression of propensity to exporting. Standard errors clustered at destination level in parentheses

Here,  $e_{onz}$  corresponds to the markup-corrected export revenue of origin country  $o$  in destination  $n$  in sector  $z$ .  $X_{on}$  is log distance and  $\hat{Z}_{onz}$  is the predicted export-pro t-to- xed-cost ratio for the highest-type firm. We include a second or third-order polynomial in this variable to proxy for the exporting firms' type mix in each origin-destination-sector combination. Finally,  $\lambda_{onz}$  is the inverse Mills ratio, which controls for unobserved variable trade costs. As usually, we first report results for this regression when the correction term is computed with  $\alpha = 5$  and  $\beta = 0$  (CRS) in Table 10. In columns (1) and (2), we report results without and with the markup correction term, but only including the inverse Mills ratio (columns labeled **Heck**), in columns (3)-(6) we instead implement the full HMR procedure. Columns (3) and (4) include a quadratic function of  $\hat{Z}_{onz}$ , while columns (5)-(6) include a third-order polynomial. In all three cases, the coefficient on distance in the specification including the markup correction term is significantly larger in magnitude compared to the one without markup correction. In our preferred specifications (columns (4)-(6)), the coefficient on log distance is around -1.284, compared to -1.15 without correction, corresponding to around 10% downward bias in absolute magnitude without correction. The corresponding point estimates of the fundamental trade cost elasticity to distance are 0.32 vs. 0.29. Finally, note that the inverse Mills ratio and the polynomial terms in  $\hat{Z}_{onz}$  have the expected signs in columns (4)-(6) and are mostly significant. Lower unobserved trade barriers (a higher inverse Mills ratio) and higher average types both increase the value of sectoral exports to a given destination.

In Table 11, we repeat the same specifications, using the mean estimated  $\alpha$  of 5.39 and  $\beta = 0$  to compute the markup correction term. Not surprisingly, our results are hardly affected by this modification. Finally, in Table 12, we use both the mean estimates of  $\alpha$  and  $\beta$  to correct for oligopolistic markups. While the point estimates on log distance hardly change, the estimated fundamental distance coefficient does become significantly larger in this

case. The reason is that we estimate significantly decreasing returns to scale for the average sector ( $\epsilon = 0$ ):

Table 11: Sector-level Gravity Estimates { intensive margin, = 5:39, = 0

Regressor	Heck w/o	Heck w/	HMR <sup>2</sup> w/o	HMR <sup>2</sup> w/	HMR <sup>3</sup> w/o	HMR <sup>3</sup> w/
log distance	-1.189*** (0.198)	-1.341*** (0.222)	-1.151*** (0.190)	-1.297*** (0.211)	-1.150*** (0.193)	-1.297*** (0.214)
inv mills	-0.121 (0.165)	-0.131 (0.184)	0.678*** (0.169)	0.810*** (0.194)	0.639** (0.309)	0.822** (0.348)
log $\hat{Z}$			0.907*** (0.265)	1.083*** (0.295)	0.736 (1.305)	1.132 (1.405)
log $\hat{Z}^2$			-0.103* (0.0565)	-0.127** (0.0626)	-0.0297 (0.534)	-0.148 (0.570)
log $\hat{Z}^3$					-0.0102 (0.0693)	0.00293 (0.0736)
$\hat{\alpha}^{distance}$	0.277	0.313	0.268	0.302	0.268	0.302
Observations	60,662	60,662	60,662	60,662	60,662	60,662
R-squared	0.302	0.272	0.304	0.274	0.304	0.274
Origin-product FE	YES	YES	YES	YES	YES	YES
Destination-product FE	YES	YES	YES	YES	YES	YES

Note: Sector-level data. Cournot model with mean of estimated  $\hat{\alpha}$  and  $\hat{\beta}$  = 0. Standard errors clustered at destination level in parentheses.

Table 12: Sector-level Gravity Estimates { intensive margin, = 5:39, = 0:34

Regressor	Heck w/o	Heck w/	HMR <sup>2</sup> w/o	HMR <sup>2</sup> w/	HMR <sup>3</sup> w/o	HMR <sup>3</sup> w/
log distance	-1.189*** (0.198)	-1.243*** (0.206)	-1.151*** (0.190)	-1.202*** (0.197)	-1.150*** (0.193)	-1.202*** (0.200)
inv mills	-0.121 (0.165)	-0.124 (0.171)	0.678*** (0.169)	0.725*** (0.178)	0.639** (0.309)	0.703** (0.322)
log $\hat{Z}$			0.907*** (0.265)	0.969*** (0.275)	0.736 (1.305)	0.876 (1.338)
log $\hat{Z}^2$			-0.103* (0.0565)	-0.112* (0.0586)	-0.0297 (0.534)	-0.0714 (0.546)
log $\hat{Z}^3$					-0.0102 (0.0693)	-0.00556 (0.0707)
$\hat{\alpha}^{distance}$	0.767	0.802	0.743	0.776	0.743	0.776
Observations	60,662	60,662	60,662	60,662	60,662	60,662
R-squared	0.302	0.292	0.304	0.294	0.304	0.294
Origin-product FE	YES	YES	YES	YES	YES	YES
Destination-product FE	YES	YES	YES	YES	YES	YES

ily biased.

Table 13: Sector-level estimates by 2-digit product.

Median est coefficient	$\alpha = 5; \beta = 0$	est, $\alpha = 0$	; est
log distance w/ corr	-1.040	-1.023	-0.930
log distance w/o corr	-0.880	-0.873	-0.872
$\hat{\alpha}_{distance}$ w/ corr	0.260	0.294	0.622
$\hat{\alpha}_{distance}$ w/o corr	0.220	0.278	0.585
abs. pct. bias (10th pctile)	4%	1.4%	0.01%
abs. pct. bias (median)	13.7 %	11.2%	% 5.2
abs. pct. bias (90th pctile)	39.9%	46.4%	19.4%

Note: Sector-level data. Median estimated coefficients by industry. Cournot model.

## 6 Monte Carlo Simulations

In this section, we perform Monte Carlo simulations to evaluate the merits of our oligopoly correction terms. To this end, we develop and calibrate a model in which firms first self-select into export destinations and then compete in quantities. Using the calibrated model, we generate a Monte Carlo data-set in which the estimation challenges (due to oligopolistic behavior and selection into export markets) discussed in Sections 2.1 and 4.1 are present. We then apply our firm- and industry-level estimators to that data-set and confirm that our oligopoly correction terms significantly improve the accuracy of our estimates.

### 6.1 Setup

The oligopoly model is as described in Section 2, with  $\alpha = 1$  (Bertrand-Nash conduct). In the following, we focus on a sector  $z$  and drop the sector index to ease notation. We now put more structure on the distribution of cost and quality shocks, as well as on how firms make

are parameters. Finally, we set  $a_{in}$  (the quality of product  $i$  in market  $n$ ) equal to 1 for every  $i$  and  $n$ .<sup>20</sup>

A country- $o$  firm that wants to sell in country  $n \neq o$  must pay a fixed cost  $f_{on} = F \frac{c_o}{c_n}$ , where  $F$  is a parameter and  $\frac{c_o}{c_n}$  and  $\frac{u_o}{u_n}$  are i.i.d. draws from a standard log-

GDP in the data. We assume that, for every country  $\alpha$ ,  $j \in \mathcal{J}_\alpha$ , the number of firms based in  $\alpha$ , is proportional to that country's GDP, with the proportionality coefficient chosen so that the total number of firms is 220. The elasticity of substitution  $\sigma$  and the returns-to-scale parameter



of the entry game using the behavioral assumption mentioned in the previous subsection, and the oligopoly equilibrium using a variant of Nocke and Schutz (2018b)'s nested fixed-point algorithm. Having done that for all ten runs, we compute arithmetic averages (or medians) across runs to obtain Monte Carlo approximations to our theoretical moments.

Our calibration algorithm converges to  $\mathbf{F} = 1.34 \cdot 10^{-9}$  (times total world expenditures in the sector, which we normalized to 000

Table 14: Monte Carlo: Firm-Level Results

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	OLS	OLS	PPML	PPML	OLS	OLS	PPML	PPML
VARIABLES	all	all	all	all	top3 exp	top3 exp	top3 exp	top3 exp
	no corr	corr	no corr	corr	no corr	corr	no corr	corr
ldist	-0.576*** (0.0777)	-0.588*** (0.0781)	-0.966*** (0.0801)	-1.446*** (0.291)	-0.624** (0.261)	-0.645** (0.276)	-1.064*** (0.146)	-1.632*** (0.451)
Observations	116,483	116,483	427,505	427,505	8,981	8,981	21,405	21,405
R-squared	0.326	0.322			0.455	0.446		
Firm-year FE	YES	YES	YES	YES	YES	YES	YES	YES
Destination-year FE	YES	YES	YES	YES	YES	YES	YES	YES

Note: True distance coefficient is 1:4.

that explicitly account for selection (columns Heckman and HMR), when combined with our correction term, deliver estimates that are very close to the true distance coefficient (1:4).

Table 15: Monte Carlo: Industry-Level Results

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	OLS	OLS	PPML	PPML	Heckman	Heckman	HMR	HMR
VARIABLES	all	all	all	all	all	all	all	all
	no corr	corr	no corr	corr	no corr	corr	no corr	corr
ldist	-1.165*** (0.0632)	-1.300*** (0.0720)	-0.966*** (0.0801)	-1.192*** (0.123)	-1.202*** (0.0748)	-1.341*** (0.0864)	-1.233*** (0.0754)	-1.378*** (0.0852)
$\log \hat{Z}$							-15.96 (25.41)	-23.24 (29.61)
$(\log \hat{Z})^2$							11.21 (15.27)	15.64 (18.00)
$(\log \hat{Z})^3$							-2.890 (3.175)	-3.869 (3.785)
inv. Mills					0.770 (0.758)	0.845 (0.878)	-1.006 (4.095)	-2.068 (4.636)
Observations	7,628	7,628	8,432	8,432	7,628	7,628	7,628	7,628
R-squared	0.703	0.668			0.703	0.668	0.703	0.668
Firm-year FE	YES	YES	YES	YES	YES	YES	YES	YES
Destination-year FE	YES	YES	YES	YES	YES	YES	YES	YES

Note: True distance coefficient is 1:4.

## 7 Conclusions

In this paper, we have evaluated the consequences of oligopolistic behavior for the estimation of gravity equations for trade flows. We showed that with oligopolistic competition, firm-

multiplied by an exporter-destination-specific Herfindahl-Hirschman index. We showed how to construct appropriate correction terms for both cases that can be used to avoid problems of omitted variable bias. Using combined French and Chinese firm-level export data as well as a sample of product-level imports by European countries, we showed that correcting for oligopolistic behavior can lead to substantial changes in the coefficients on standard gravity regressors.

## Appendix

### A Proofs

#### A.1 Proof of Proposition 1

**Proof.** To complete the proof of the proposition, we need to: (a) Show that the function  $S$  is well defined, and study its monotonicity properties as well as its limits; (b) show that the equilibrium condition (10) has a unique solution; (c) show that, at  $\alpha = 1$ , the first-order

(c) Rewriting equation (2) with  $\alpha = 1$  and rearranging terms yields:

$$\frac{\partial \pi_i}{\partial q_j} = q_j^2 \left[ \frac{1}{E} \frac{a_j q_j^{1+\alpha}}{a_j q_j^{1-\alpha}} \right] - \frac{1}{A} \frac{a_j q_j^{1-\alpha}}{a_j q_j^{1-\alpha}} c_j q_j^3;$$

where we have dropped the destination subscript for ease of notation. As  $1 + \alpha > 0$ , the term inside square brackets is strictly decreasing in  $q_j$ . Moreover, that term tends to  $+\infty$  and  $-c_j q_j^3$  as  $q_j$  tends to 0 and  $+\infty$ , respectively. It follows that  $q_j$  maximizes firm  $i$ 's profit if and only if firm  $i$ 's first-order condition holds at  $q_j$ .  $\square$

## A.2 Proof of Proposition 2

**Proof.** To apply Taylor's theorem, we require the value of  $\mathbf{s}_e^0(0)$ . This requires computing the partial derivatives of  $\mathbf{S}(\cdot; \cdot)$  at  $\alpha = 0$  as well as  $\mathbf{H}^0(0)$ . Differentiating equation (9) with respect to  $s_i$ ,  $\alpha$ , and  $t_i$  at  $\alpha = 0$  yields

$$s_i d = \frac{1 + \frac{ds_i}{s_i}}{1} - \frac{(1 + \alpha) d_i}{1}$$

We can now compute  $\mathbf{s}_i^0(0)$ :

$$\begin{aligned} \mathbf{s}_i^0(0) &= \frac{\partial \mathbf{S}}{\partial \mathbf{H}} \frac{\mathbf{T}_i}{\mathbf{H}(\cdot)}; \\ &= \frac{\mathbf{T}_i}{\mathbf{H}(0)} \frac{\partial \mathbf{S}}{\partial \mathbf{H}} \frac{\mathbf{T}_i}{\mathbf{H}(0)}; \quad \frac{\mathbf{H}^0(0)}{\mathbf{H}(0)} + \frac{\partial \mathbf{S}}{\partial \mathbf{H}} \frac{\mathbf{T}_i}{\mathbf{H}(0)}; \\ &= \frac{1}{1 + \mathbf{s}_i(0) \text{HHI}(0)} (\mathbf{s}_i(0))^2 : \end{aligned}$$

It follows that

$$\begin{aligned} \frac{\mathbf{s}_e^0(0)}{\mathbf{s}_e(0)} &= \frac{1}{1 + \sum_{j \in E} \mathbf{s}_j(0) \text{HHI}(0)} \sum_{j \in E} \mathbf{s}_j(0)^2 \\ &= \frac{1}{1 + \text{HHI}(0)} \sum_{j \in E} \frac{\mathbf{s}_j(0)^2}{\mathbf{s}_e(0)} \\ &= \frac{1}{1 + \text{HHI}(0)} [\text{HHI}(0) \sum_{j \in E} \mathbf{s}_j(0) \text{HHI}_e(0)]: \end{aligned}$$

Applying Taylor's theorem at the first order in the neighborhood of  $\mathbf{s}_e = 0$  yields:

$$\begin{aligned} \log \mathbf{s}_e(\cdot) &= \log \mathbf{s}_e(0) + \frac{d}{d\mathbf{s}_e} \log \mathbf{s}_e(\cdot) \Big|_{\mathbf{s}_e=0} + o(\cdot) \\ &= \log \mathbf{s}_e(0) + \frac{1}{1 + \text{HHI}(0)} [\text{HHI}(0) \sum_{j \in E} \mathbf{s}_j(0) \text{HHI}_e(0)] + o(\cdot) \\ &= \log \mathbf{s}_e(0) + \frac{1}{1 + \text{HHI}(0)} [\text{HHI}(\cdot) \sum_{j \in E} \mathbf{s}_j(\cdot) \text{HHI}_e(\cdot)] + o(\cdot); \end{aligned}$$

where the last line follows from the fact that  $\text{HHI}(\cdot) - \text{HHI}(0)$  and  $\sum_{j \in E} \mathbf{s}_j(\cdot) \text{HHI}_e(\cdot) - \sum_{j \in E} \mathbf{s}_j(0) \text{HHI}_e(0)$  are at most first order.  $\square$

## B Price Competition

### B.1 Theoretical Results

Under price competition, the profit of firm  $i$  when selling in destination  $n$  is:

$$\pi_{in} = \mathbf{p}_{in} \mathbf{a}_{in} \mathbf{p}_{in}^{-1} \mathbf{P}_n^{-1} \mathbf{E}_n - \mathbf{C}_{in} \mathbf{a}_{in} \mathbf{p}_{in}^{-1} \mathbf{P}_n^{-1} \mathbf{E}_n;$$

where we have dropped the sector index  $\mathbf{z}$  for ease of notation.

The degree of strategic interactions between firms continues to be governed by the conduct

parameter  $\alpha \in [0; 1]$ : When firm  $i$  increases its price by an infinitesimal amount, it perceives the induced effect on  $P_n$  to be equal to  $\frac{\partial P_n}{\partial p_i} = \alpha \frac{P_n}{p_i}$ . It is still the case that monopolistic competition arises when  $\alpha = 0$ , whereas Bertrand competition arises when  $\alpha = 1$ . The first-order condition of profit maximization of firm  $i$  in destination  $n$  is given by

$$0 = \frac{\partial \pi_{in}}{\partial p_i} = a_{in} p_{in}^{-1} P_n^{-1} E_n + (p_{in} - C_{in}^0(q_{in})) \frac{1}{p_{in}} + \frac{1}{P_n} \frac{\partial P_n}{\partial p_i} E_n a_{in} p_{in}^{-1} \\ = q_{in}^{-1} \frac{p_{in} - C_{in}^0(q_{in})}{p_{in}} [1 - (\alpha - 1) s_{in}] ; \quad (15)$$

where

$$s_{in} = \frac{a_{in} p_{in}^1}{\sum_{j \in J} a_{jn} p_{jn}^1} \quad (16)$$

continues to be the market share of firm  $i$  in destination  $n$ .

Equation (15) pins down firm  $i$ 's optimal markup under price competition:

$$m_{in} = \frac{1}{(\alpha - 1) s_{in}} ;$$

where  $m_{in} = \frac{p_{in} - C_{in}^0(q_{in})}{p_{in}}$  is firm  $i$ 's Lerner index. Apart from this change in the expression for the firm's optimal markup, all other firm-level results go through as before.

We now turn our attention to the sector-level results. As in Section 4, we begin by employing an aggregative games approach to analyze the equilibrium in a given market, dropping the market subscript  $n$  to ease notation. The market-level aggregator  $H$  is now defined as

$$H = P^1 = \sum_{j \in J} a_j p_j^1$$

and firm  $i$ 's type as

$$T_i = a_i (E)^{\frac{(1-\alpha)}{1+\alpha}} (c_i)^{\frac{1}{1+\alpha}} ;$$

Plugging these definitions into equation (15), making use of equation (16), and rearranging, we obtain:

$$1 - s_i^{\frac{1+\alpha}{\alpha}} \frac{H}{T_i} \frac{1+\alpha}{\alpha} [1 - (\alpha - 1) s_i] = 1 : \quad (17)$$

Note that the left-hand side of equation (17) is strictly decreasing on the interval

$$\left( \frac{1}{\alpha - 1} ; \frac{T_i}{H} \frac{1+\alpha}{\alpha} \right) ;$$

and tends to  $\infty$  and 0 as  $s_i$  tends to the lower and upper endpoints of that interval, respectively. Equation (17) therefore has a unique solution on the above interval, denoted  $\mathbf{S}(t_i; \tau)$  with  $t_i = T_i/H$ . (Solutions outside that interval necessarily give rise to strictly negative markups and are thus suboptimal.)

It is easily checked that  $\mathbf{S}$  is strictly increasing in its first argument, strictly decreasing in its second argument, and tends to 0 and 1 as  $t_i$  tends to 0 and  $\tau$ , respectively.

As before, the equilibrium condition is that market shares must add up to unity:

$$\sum_{j=2}^J \mathbf{S} \left( \frac{T_j}{H}; \tau \right) = 1: \quad (18)$$

The properties of the function  $\mathbf{S}$ , described above, imply that this equation has a unique solution,  $H(\tau)$ .

To summarize:

**Proposition A.** In each destination market, and for any conduct parameter, there exists a unique equilibrium in prices. The equilibrium aggregator level  $H(\tau)$  is the unique solution to equation (18). Each firm  $i$ 's equilibrium market share is  $s_i(\tau) = \mathbf{S}(T_i/H(\tau); \tau)$ , where  $\mathbf{S}(T_i/H(\tau); \tau)$  is the unique solution to equation (17). From equation (16), firm  $i$ 's equilibrium price is given by

$$p_i(\tau) = \frac{s_i(\tau) H(\tau)^{\frac{1}{1+\tau}}}{a_i}:$$

**Proof.** All that is left to do is check that first-order conditions are sufficient for optimality when  $\tau = 1$ . Combining equations (15) and (17) yields:

$$\frac{\partial \pi_i}{\partial p} = q_i [1 - (p_i)^\tau (p_i)];$$

where

$$(p_i)^\tau = 1 - \frac{p_i^{\tau+1} a_i}{\sum_j a_j p_j^{\tau+1}} \quad \text{and} \quad (p_i)^\tau = \left( \frac{p_i}{H(\tau)} \right)^{\tau+1} \frac{a_i p_i^{\tau+1}}{\sum_j a_j p_j^{\tau+1}}:$$

As  $1 + \tau > 0$ , the functions  $(p_i)^\tau$  and  $(p_i)^\tau$  are strictly increasing. Moreover,  $(p_i)^\tau > 0$  for every  $p_i$ , whereas there exists  $\underline{p}_i > 0$  such that  $(p_i)^\tau > 0$  if  $p_i > \underline{p}_i$  and  $(p_i)^\tau < 0$  if  $p_i < \underline{p}_i$ . Hence,  $\pi_i$  is strictly increasing on the interval  $(0; \underline{p}_i)$ , and firm  $i$ 's first-order condition holds nowhere on that interval. The fact that  $\lim_{p_i \rightarrow 1} (p_i)^\tau = 1$  and  $\lim_{p_i \rightarrow 0} (p_i)^\tau = 0$  and the

monotonicity properties of  $\mathbf{s}_e$  and  $\mathbf{s}_i$  on  $(\mathbf{p}_i; \tau)$  imply the existence of a unique  $\mathbf{p}_i$  at which firm  $i$ 's first-order condition holds. Moreover,  $\mathbf{s}_i$  is strictly increasing on  $(\mathbf{p}_i; \mathbf{p}_i)$  and strictly decreasing on  $(\mathbf{p}_i; \tau)$ . First-order conditions are therefore sufficient for optimality.  $\square$

Having characterized the equilibrium in a given destination, we now adapt the first-order approach to sector-level gravity to the case of price competition. As in Section 4, let  $E(\mathcal{J})$  denote the subset of exporters in country  $e$  that sell in the destination market  $n$ . The combined market share of those exporters in market  $n$  is given by

$$\mathbf{s}_e(\tau) = \sum_{i \in E(\mathcal{J})} \mathbf{s}_i(\tau):$$

As before, we approximate  $\mathbf{s}_e(1)$  at the first order. The definitions of HHI and  $\text{HHI}_e$  are as in Section 4.

We obtain:

**Proposition B.** At the first order, in the neighborhood of  $\tau = 0$ , the logged joint market share in destination  $n$  of the firms from export country  $e$  is given by

$$\log \mathbf{s}_e(\tau) = \log \mathbf{s}_e(0) + \frac{1}{(1 + \tau)} [\text{HHI}(\tau) - \mathbf{s}_e(\tau) \text{HHI}_e(\tau)] + o(\tau):$$

**Proof.** The proof follows the same developments as the proof of Proposition 2. We begin by computing the partial derivatives of  $\mathbf{S}$  at  $\tau = 0$ . It is useful to rewrite first equation (17) as

$$\mathbf{s}_i = \mathbf{t}_i^{\frac{1+\tau}{1-\tau}} \left( 1 - \frac{1}{(1 + \tau)\mathbf{s}_i} \right)^{\frac{1}{1+\tau}}: \quad (19)$$

Taking the logarithm and totally differentiating the equation at  $\tau = 0$  yields:

$$\frac{d\mathbf{s}_i}{\mathbf{s}_i} = \frac{1 + \tau}{1 + \tau} \frac{d\mathbf{t}_i}{\mathbf{t}_i} - \frac{1}{(1 + \tau)} \mathbf{s}_i d \left( \frac{1}{(1 + \tau)\mathbf{s}_i} \right):$$

The partial derivatives of  $\mathbf{S}$  are thus given by





Table 16: Price Elasticities and Returns-to-Scale Estimates { Price Competition

Mean	4.96	0.31
25th Percentile	2.06	0.02
Median	3.27	0.10
75th Percentile	5.22	0.28
Min	1.01	-0.11
Max	26.03	4.5
Standard Deviation	4.89	0.67
HS 2-digit products	78	78

Note: Table shows descriptive statistics for estimates of  $\epsilon$  and  $\eta$ . Estimates computed using 6-digit HS firm-level information but constrained to be identical within 2-digit HS products.

Tables 17-19 show results for the pooled firm-level regressions. In all specifications, the point estimates on the distance coefficient are much larger in absolute magnitude when correcting for oligopoly bias. The absolute value of the distance coefficient is slightly smaller than with Cournot competition.

Tables 20-22 show results for the pooled sector-level regressions. Again, the distance coefficient becomes larger in absolute magnitude when including the markup correction term. Like in the case of Cournot competition, the absolute differences in coefficient magnitudes between the estimates with and without correction are smaller than with the firm-level estimates.

Table 17: Firm-level Gravity Estimates { Bertrand competition,  $\alpha = 5$ ,  $\beta = 0$ .

Regressor	PPML w/ corr	PPML w/o corr	OLS w/ corr	OLS w/o corr
log distance	-1.418*** (0.190)	-0.874*** (0.021)	-0.246*** (0.014)	-0.232*** (0.014)
$\hat{\alpha}_{\text{distance}}$	0.355	0.219		-
Observations	11,955,786	11,955,786	708,386	708,386
R-squared			0.05	0.06
Firm-year FE	YES	YES	YES	YES
Product-dest.-year FE	YES	YES	YES	YES

Note: Firm-level data, pooled across sectors. Results for top 3 exporters. Bertrand model with  $\alpha = 5$  and  $\beta = 0$ . Standard errors in brackets, clustered at the destination-year level.

Table 18: Firm-level Gravity Estimates { Bertrand competition,  $\alpha = 4.96$ ,  $\beta = 0$ .

Regressor	PPML w/ corr	PPML w/o corr	OLS w/ corr	OLS w/o corr
log distance	-1.415*** (0.189)	-0.874*** (0.0210 )	-0.246*** (0.013)	-0.232*** (0.013)
$\hat{\alpha}_{distance}$	0.357	0.221		
Observations	11,955,786	11,955,786	708,392	708,386
R-squared			0.06	0.06
Firm-year FE	YES	YES	YES	YES
Product-dest.-year FE	YES	YES	YES	YES

Note: Firm-level data, pooled across sectors. Results for top 3 exporters. Bertrand model with mean of estimated  $\alpha$  and  $\beta = 0$ . Standard errors in brackets, clustered at the destination-y

Table 21: Sector-level Gravity Estimates { Bertrand Competition,  $\alpha = 4:96$ ,  $\beta = 0$

Regressor	Heck w/o	Heck w/	HMR <sup>2</sup> w/o	HMR <sup>2</sup> w/	HMR <sup>3</sup> w/o	HMR <sup>3</sup> w/
log distance	-1.189*** (0.198)	-1.217*** (0.202)	-1.151*** (0.190)	-1.177*** (0.194)	-1.150*** (0.193)	-1.177*** (0.197)
inv mills	-0.121 (0.165)	-0.123 (0.168)	0.678*** (0.169)	0.702*** (0.173)	0.639** (0.309)	0.672** (0.315)
log $\hat{Z}$			0.907*** (0.265)	0.939*** (0.270)	0.736 (1.305)	0.808 (1.322)
log $\hat{Z}^2$			-0.103* (0.0565)	-0.108* (0.0575)	-0.0297 (0.534)	-0.0512 (0.540)
log $\hat{Z}^3$					-0.0102 (0.0693)	-0.00780 (0.0700)
$\hat{distance}$						
Observations	60,662	60,662	60,662	60,662	60,662	60,662
R-squared	0.302	0.297	0.304	0.299	0.304	0.299
Origin-product FE	YES	YES	YES	YES	YES	YES
Destination-product FE	YES	YES	YES	YES	YES	YES

Note: Sector-level data. Bertrand model with mean of estimated  $\alpha$  and  $\beta = 0$ . Standard errors clustered at destination level in parentheses.

Table 22: Sector-level Gravity Estimates { Bertrand competition,  $\alpha = 4:96$ ,  $\beta = 0:31$

Regressor	Heck w/o	Heck w/	HMR <sup>2</sup> w/o	HMR <sup>2</sup> w/	HMR <sup>3</sup> w/o	HMR <sup>3</sup> w/
log distance	-1.189*** (0.198)	-1.200*** (0.199)	-1.151*** (0.190)	-1.161*** (0.192)	-1.150*** (0.193)	-1.161*** (0.194)
inv mills	-0.121 (0.165)	-0.122 (0.166)	0.678*** (0.169)	0.687*** (0.171)	0.639** (0.309)	0.652** (0.311)
log $\hat{Z}$			0.907*** (0.265)	0.919*** (0.267)	0.736 (1.305)	0.765 (1.311)
log $\hat{Z}^2$			-0.103* (0.0565)	-0.105* (0.0569)	-0.0297 (0.534)	-0.0382 (0.536)
log $\hat{Z}^3$					-0.0102 (0.0693)	-0.00925 (0.0696)
$\hat{distance}$						
Observations	60,662	60,662	60,662	60,662	60,662	60,662
R-squared	0.302	0.300	0.304	0.302	0.304	0.302
Origin-product FE	YES	YES	YES	YES	YES	YES
Destination-product FE	YES	YES	YES	YES	YES	YES

Note: Sector-level data. Bertrand model with mean of estimated  $\alpha$  and  $\beta = 0.31$ . Standard errors clustered at destination level in parentheses.

## C Data Appendix

Table 23: List of export destinations included in the firm-level and product-level data

Austria	Latvia
Belgium	Lithuania
Bulgaria	Luxembourg
Croatia	Malta
Cyprus	Netherlands
Czech Rep.	Norway
Denmark	Poland
Estonia	Portugal
Finland	Romania
France	Serbia
Germany	Slovakia
Greece	Slovenia

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